Learning outcomes: Chapter 3

1. You should know how to obtain parameters for a prior distribution for θ given some suggested prior summaries for θ. For example,

   – parameters for the gamma distribution for θ given $E(\theta)$ and $Var(\theta)$;

   – parameters for the beta distribution for θ given the mode for θ and a probability for θ.

2. You should be able to describe the *trial roulette* and *bisection* methods of prior elicitation. You should also be able to:

   – Interpret output from the *MATCH* elicitation tool;

   – explain the processes of *feedback* and *refinement*;

   – explain the role of *feedback percentiles*, and be able to transpose these into suitable questions to ask a non–statistician.

3. You should be able to define *substantial prior information*.

4. You should be able to explain how we might operate Bayes' theorem with *vague prior knowledge*, and you should be able to obtain posterior distributions in circumstances when our prior knowledge about θ is indeed vague.

5. You should be able to explain *prior ignorance*; you should also memorise the form of Jeffreys' prior for θ:

$$\pi(\theta) \propto \sqrt{I(\theta)}$$

   where

$$I(\theta) = E_{\mathbf{X}|\theta}\left[-\frac{\partial^2}{\partial\theta^2}\log f(\boldsymbol{X}|\theta)\right],$$

   and be able to obtain this prior for a given model $f(\boldsymbol{x}|\theta)$.

6. You should be able to state, and provide an outline proof of, the asymptotic posterior distribution for θ:

$$\theta|\boldsymbol{x} \sim N\left(\hat{\theta}, J(\hat{\theta})^{-1}\right) \qquad \text{approximately,}$$

   where $J(\theta)$ is the *observed information*

$$J(\theta) = -\frac{\partial^2}{\partial\theta^2}\log f(\boldsymbol{X}|\theta).$$

7. Given a model $f(\boldsymbol{x}|\theta)$, you should be able to obtain the asymptotic posterior distribution for θ.