MAS2903: Introduction to Bayesian Methods

Dr. Lee Fawcett

Case Study 2: Bayesian Modelling of Extreme Rainfall Data

Semester 2, 2019—2020

Motivation

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of environmental phenomena resulting in huge economic loss, and loss of human life.

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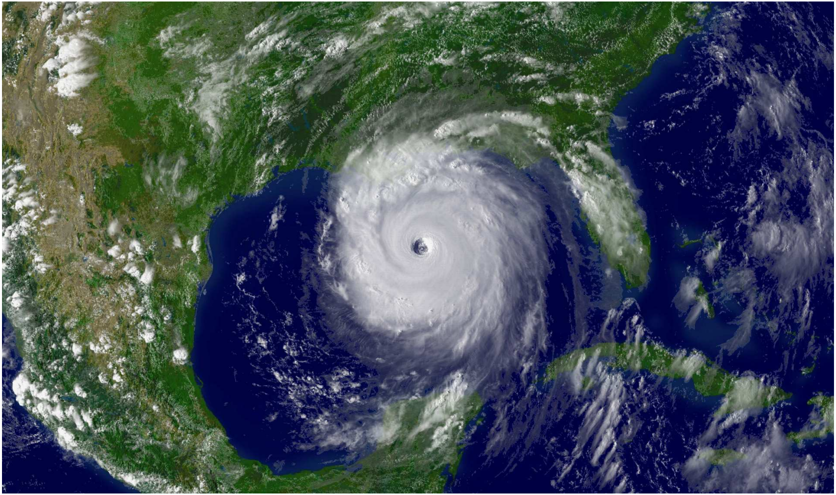
Sea–surge: Hurricane Katrina, 2005



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Wind damage from UK storms

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- Loss of life, huge economic burden, including massive flood insurance premiums

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Rainfall: Flooding in Central England, 2008

Rainfall: Flooding in Newcastle, June 2012

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Rainfall: The Great North Sea Flood, 1953

Rainfall: The Great North Sea Flood, 2025?

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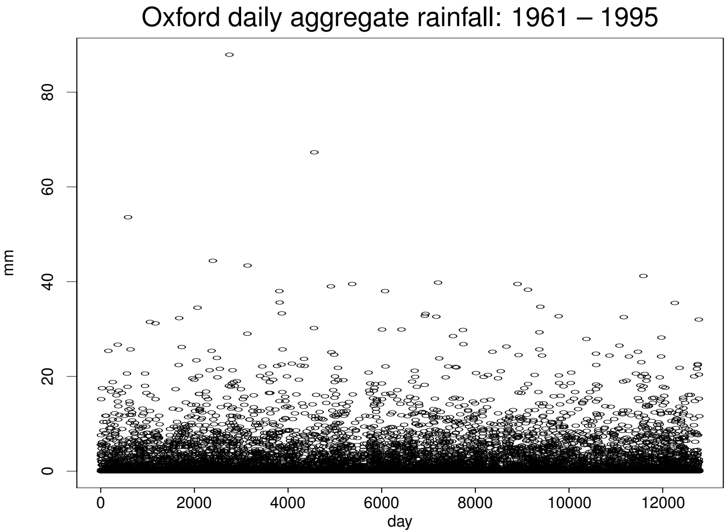
  - Nearly 13,000 observations for each site!
  - However, not interested in most of them – e.g. zero values or indeed anything **non–extreme**!
- **Idea:** Extract annual maxima!

Statistical modelling of extreme rainfall data



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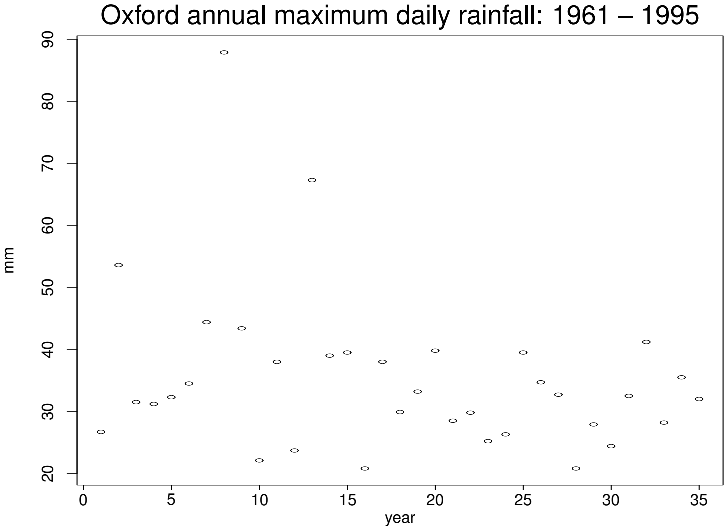

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- Can use the extracted annual maxima!

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$$\hat{\mu} = 40.8(1.58) \qquad \hat{\sigma} = 9.7(1.19) \qquad \hat{\gamma} = 0.1(0.11)$$

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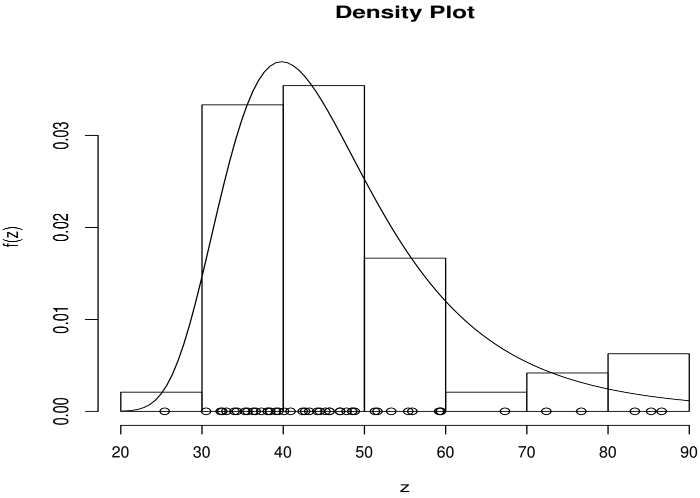

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- We can estimate such quantities by calculating **high quantiles** from our fitted distribution.

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- The height of the flood defence would be a function of $\hat{z}_{100}$ and the duration of the storm event

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r (years)	**10**	**50**	**200**	**1000**
$\hat{z}_r$	65.54 (4.53)	87.92 (11.48)	98.64 (16.22)	140.34 (41.83)

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  - But it can also reduce estimation uncertainty!

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  - ... perhaps easier for μ?

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Idea: Re–express our GEV in terms of parameters Dr. Reede will feel comfortable with – perhaps **return levels**!

"What sort of daily rainfall accumulation would you expect to see, at Oxford, in a storm that might occur once in ten years?"

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"... 60mm–65mm? Range might be 50mm$\longrightarrow$80mm..."

Can use the *MATCH* tool to hep here:

http://optics.eee.nottingham.ac.uk/match/uncertainty.php

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$$
\begin{aligned}
z_{10} &\sim Ga(126, 2) \qquad \text{and} \\
z_{50} &\sim Ga(242, 2.5) \\
z_{200} &\sim Ga(180, 1.5)
\end{aligned}
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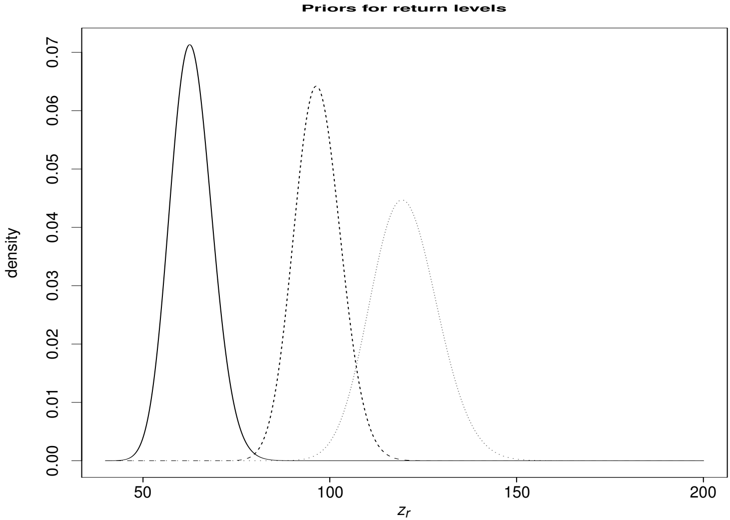

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Obtaining the posterior distribution

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- Use **Markov chain Monte Carlo** (MCMC – see *MAS3902: Bayesian Inference*)
- This gives a sample from the posteriors for μ, σ and γ
- Apply **Equation (3)** to obtain posterior distribution for return levels

Results

r (years)	**10**	**50**	**200**	**1000**
$E(\hat{z}_r \mid \boldsymbol{x})$	64.21 (2.14)	91.05 (6.31)	110.31 (8.05)	150.73 (14.79)
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r (years)	**10**	**50**	**200**	**1000**
Bayesian	(60.0,68.4)	(78.7,103.4)	(94.5,126.1)	(121.7,179.7)
Frequentist	*(56.7,74.4)*	*(65.4,110.4)*	*(66.8,130.5)*	*(58.3,222.4)*

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- The Statistician then feeds back their results to the Marine Engineers designing the flood defence system – they usually build to a height specified by the upper endpoint of a **95% Bayesian confidence interval** for $\hat{z}_r$!