MAS2903: Introduction to Bayesian Methods

Dr. Lee Fawcett

Case Study 1: Speed Cameras and Regression to the Mean

Semester 2, 2019-2020

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- 24,690 seriously injured
- Huge economic and human cost on society

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"Speed Cameras reduce road deaths by 70%"

Similar messages were reported in the local/national media praising the use of speed cameras:

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Paul Smith, founder of SafeSpeed: Speed cameras:

- "Are just another government tax"
- "Do not account for Regression to the Mean"
 - "Are no more effective than lots of garden gnomes



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For example, the Cleveland Safety Camera Partnership had identified 72 "**blackspot**" locations which had observed at least 5 serious casualties over a two year observation period (2000-2002).

At each of these sites a speed camera was installed, and then the number of casualties was observed over a two year treatment period (2002-2004).

The results seemed conclusive:

- In the "before" period, there were 361 casualties...
- …in the "after" period, there were 108 casualties…

...giving the 70% reduction quoted in the Hartlepool Mail

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Organisations like SafeSpeed claimed such before–after studies were flawed, as they did not account for the phenomenon of **Regression To the Mean**, or RTM.

In fact, they claimed that such studies **exaggerated the** effectiveness of speed cameras.

Indeed, they had the support of leading statisticians and RTM made the headlines of leading national newspapers.

The **Northumbria Safety Camera Partnership** was formed in 2001.

In 2004 I was asked to provide statistical support in their assessment of the effectiveness of speed cameras across the northeast.

"We'd like you to produce some statistics that prove that speed cameras save lives and reduce the financial burden of road traffic accidents to the NHS"

"Teesside SCP only quoted percentage changes... we'd like some hard statistics, maybe paired t-tests and the like" The **Northumbria Safety Camera Partnership** was formed in 2001.

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I felt uncomfortable about this.

- I was clocked by a speed camera three days after passing my driving test in 2002
 - I was doing 32mph in a 30 zone
 - I got 3 points
 - I got a £60 fine
- Hard statistics: "*t*-tests and the like"??

Then I realised how much statisticians can charge for their services!

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Regression To the Mean (RTM)



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In particular, I thought about RTM and how we could account for this.

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- Take ten dice
- Roll each twenty times noting the number of sixes for each
- Place a piece of paper under the two "highest scoring" dice
- Roll each of these dice another twenty times on top of the paper and count the number of sixes observed again
- The second total is almost always lower than the first, proving that the piece of paper decreased the number of sixes rolled by the dice

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- Before: 12
- After: 3
- 75% reduction!

| Student | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---------|---|---|---|---|---|----|---|----|---|----|
| Sixes | 3 | 3 | 1 | 1 | 0 | 7* | 4 | 5* | 2 | 2 |
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So, the argument is that

- because speed cameras are installed at sites with an abnormally high casualty record
- these sites would probably see a reduction in casualties in the after period anyway
- even without the cameras!

Perhaps the cameras do have a role to play, but we shouldn't really attribute the *entire* reduction to the cameras.

Some of this might have happened anyway!

Ideal world: *randomly* choose some sites for speed cameras. Ethical concerns...

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- **Ideal world:** *randomly* choose some sites for speed cameras. Ethical concerns...
- **Even more ideal world:** *randomly* choose some sites for speed cameras then observe what happens in a "parallel universe"... not practical.

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154 casualties at 17 blackspots in South Tyneside in a "before" period

- 41 casualties in the "after" period
- Rather than just compare before and after figures, I thought I would try to account for RTM – using a Bayesian analysis!

- 154 casualties at 17 blackspots in South Tyneside in a "before" period
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Suppose we assume that Y_i , i = 1, ..., 17 represent casualty frequencies at each blackspot site in South Tyneside.

Assume that

Let y_i be the (abnormally high) observed number of casualties from these Poisson distributions in the observation period.

How can we estimate the number of casualties that we would *normally* expect to see at the speed camera sites?
Suppose we assume that Y_i , i = 1, ..., 17 represent casualty frequencies at each blackspot site in South Tyneside.

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 $Y_i \sim Po(\theta_i)$

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- The Gamma is the conjugate prior for the Poisson (see Chapter 3)
- The gamma is defined over the positive real line
- Constant gamma "shape" g for all speed camera sites
- Site-specific gamma "rate" h_i
- How do we choose (g, h_i) ?

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Northumbria police recorded several variables at a total of 84 sites in South Tyneside

- Let's call the other 67 (non-camera) site our reference sites
- As these were not chosen for speed cameras, their casualty frequency was more "normal"
- A simple linear regression analysis gives the following for the reference sites:

$$Y_j = -4.288 + 0.157X_j + \epsilon_j,$$

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Idea: Use the regression equation obtained at the reference sites on average observed vehicle speeds at our blackspot sites to predict more "normal" casualty frequencies here!

Thus, for each speed camera site *i*, *i* = 1, . . . , 17, we obtain $\mu_i = -4.288 + 0.157 X_i.$

Recall the gamma prior for θ_i : $\theta_i \sim Ga(g, h_i)$.

Using $h_i = g/\mu_i$ gives

• $E(\theta_i) = \mu_i$ and

• $Var(\theta_i) = \mu_i^2/g$

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Obtaining the posterior for θ_i

Recall from lectures that

posterior \propto prior \times likelihood

We have a $Ga(g, g/\mu_i)$ prior for θ_i and a Poisson likelihood.

This gives (see Assignment 2)

$$heta_i | y_i \sim Ga\left(g + y_i, \frac{g}{\mu_i} + 1\right),$$

with posterior mean

$$E(\theta_i|y_i) = \alpha_i \mu_i + (1 - \alpha_i) y_i,$$

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This gives (see Assignment 2)

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- It does so using an estimate of casualty frequency we might "usually" expect to see at the blackspot sites
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- our prior beliefs about what we might normally expect to see at these blackspot sites (using the reference sites)
- with our observed (abnormally high) value
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Recall also that this gives the gamma posterior

 $\theta_i | y_i \sim Ga\left(g + y_i, \frac{g}{\mu_i} + 1\right),$

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The average speed in the before period at site 1 was $x_1 = 55$ mph, and we observed $y_1 = 20$ casualties.

Substituting this into our predictive accident model obtained from the reference set gives

 $\mu_1 = -4.288 + 0.157 \times 55 = 4.347$ casualties.

$$\theta_1 \sim Ga(g = 2.5, h_1 = 2.5/4.347)$$
 i.e.
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| Site | Уi | μ_i | α_i | $E(\theta_i \mathbf{y}_i)$ | y i,after | Difference | |
|-------|-----|---------|------------|----------------------------|------------------|------------|-----------|
| | | | | | | Observed | After RTM |
| 1 | 20 | 4.35 | 0.36 | 14.29 | 0 | -20 | -14.29 |
| : | : | : | : | : | : | : | : |
| 10 | 8 | 2.31 | 0.52 | 5.04 | 3 | -5 | -2.04 |
| : | : | ÷ | ÷ | ÷ | : | : | : |
| Total | 154 | | | 90 | 41 | -113 | -49 |

- Reduction from 154 → 90:"would have happened anyway"
- Remaining reduction from $90 \rightarrow 41$: speed cameras
- Without RTM: 73% reduction: exaggerates the effect of the cameras!
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