Test-style questions

1. Suppose $X \sim N(0,1)$ and that $Y = X^2$. Which option, A–E, correctly specifies the probability density function of Y?

A. $f_Y(y) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}y}$ for $0 \leq y < \infty$

B. $f_Y(y) = \frac{1}{2\pi} e^{-\frac{1}{4}y^2}$ for $0 \leq y < \infty$

C. $f_Y(y) = \frac{1}{\sqrt{8\pi y}} e^{-\frac{1}{2}y}$ for $0 <\leq y < \infty$

D. $f_Y(y) = \frac{1}{2^{1/2} y^{1/2} \Gamma(1/2)} e^{-\frac{1}{2}y}$ for $0 \leq y < \infty$

E. $f_Y(y) = \lambda e^{(y - \lambda e^y)}$ for $-\infty < y < \infty$

2. Suppose $X \sim Exp(2)$ and that

$$f_{Y|X}(Y|X = x) = \begin{cases} \frac{e^{-y/x}}{x}, & \text{when } y \geq 0, \\ 0, & \text{otherwise.} \end{cases}$$

Write an R function to generate 500 pairs of realizations of (X, Y). Produce a scatterplot of your realizations of X against thoser for Y, and obtain the correlation between X and Y.

In your solutions, include (i) your main function; (ii) your scatterplot; (iii) the correlation. No other code or solutions are needed.

3. Suppose

$$\begin{pmatrix} X \\ Y \end{pmatrix} \sim N_2 \left[\begin{pmatrix} 4 \\ 1 \end{pmatrix}, \begin{pmatrix} 8 & 2 \\ 2 & 5 \end{pmatrix} \right].$$

The random variable Z is defined by $Z = X + 3Y$.

(a) What is the distribution of Z?

(b) Use R to simulate 1,000 pairs of observations on (X, Y).

(c) Use your sample in (b) to to estimate $P(Z > 8)$.

(d) Use your sample in part (b) to estimate the correlation between X and Y. How does this compare to the true correlation?

4. It is suggested that the survival times of patients (X years) receiving a new cancer drug will follow a Weibull distribution with CDF

$$F_X(x; \lambda, \kappa) = 1 - e^{-(x/\lambda)^\kappa}, \quad x \geq 0, \quad \lambda, \kappa > 0,$$

where λ is referred to as the scale parameter and κ is the shape parameter.

(a) Write an R function that uses the inverse CDF method to generate a sample of size `N` from the Weibull distribution. Your function should have *three* input variables: `N`, `lambda` and `kappa`.

(b) Past studies from a similar drug indicate that $\lambda = 2$ and $\kappa = 1.5$. Use these values, and your function in part (a), to simulate 10,000 realisations from X, and plot a histogram of these values.