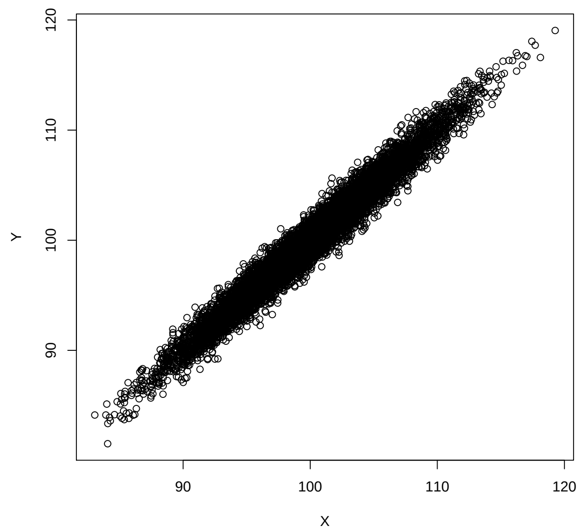
Solutions: Practical 4

1. Recognise that $Y|X = x \sim N(\mu = x, \sigma^2 = 1)$. Then:

```
Q1 = function(N){
  output = matrix(0, 2, N)
  x = rnorm(N, 100, 5)
  y = rnorm(N, x, 1)
  output[1, ] = x
  output[2, ] = y
  return(output)}
```

```
test = Q1(1000)
plot(test[1,], test[2,], xlab="X", ylab="Y")
cor(test[1,], test[2,])
[1] 0.980935
```



2. (a) We have $\mu_x = 1$; $\mu_y = 3$; $\sigma_x = \sqrt{2}$; $\sigma_y = \sqrt{1}$

   (b) $\rho = \text{cov}(X,Y)/\sigma_x\sigma_y = -0.8\sqrt{2}/\sqrt{2}\sqrt{1} = -0.8$ (or just recognize from form of variance matrix)

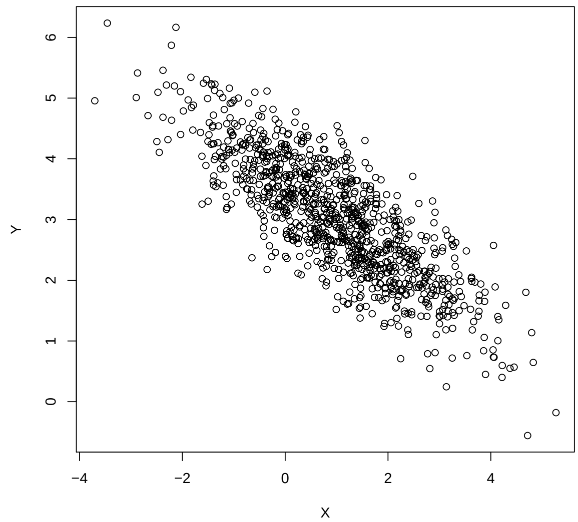
   (c) From property (2) on page 25 of the lecture notes, we have

$$Y|X = x \sim N\left(3 + -0.8\frac{\sqrt{1}}{\sqrt{2}}(x-1), 1^2(1-0.64)\right) \equiv N(3 - 0.5656854(x-1), 0.36).$$

   Then:

```r
Q2 = function(N){
  output = matrix(0, 2, N)
  x = rnorm(N, 1, sqrt(2))
  y = rnorm(N, 3-0.5656854*(x-1), sqrt(0.36))
  output[1, ] = x
  output[2, ] = y
  return(output)}

test = Q2(1000)
plot(test[1,], test[2,], xlab="X", ylab="Y")
cor(test[1,], test[2,])
[1] -0.8164408
```




```r
library(MASS)
npts = 1000
mu = c(1, 3)
sigma = matrix(data = c(2, -0.8*sqrt(2), -0.8*sqrt(2), 1), nrow=2,
    byrow=TRUE)
test2 = mvrnorm(npts, mu, sigma)
plot(test2[,1], test2[,2])
cor(test2[,1], test2[,2])
[1] -0.7970329
```