Solutions: Practical 3

1. (a) Code:

```
Y1 = sqrt(runif(1000, 1, 5))
Y2 = log(runif(1000, 1, 5))
```

(b) For $Y1$:

$$F_{Y1}(y) = \Pr(Y_1 \leq y) = \Pr\left(\sqrt{X} \leq y\right) = \Pr(X \leq y^2) = F_X(y^2).$$

Now

$$\begin{aligned} f_{Y1}(y) &= \frac{d}{dy} F_X(y^2) \\ &= f_X(y^2) \times 2y \quad \text{by chain rule} \\ &= 0.25 \times 2y = 0.5y, \quad \sqrt{1} \leq y \leq \sqrt{5}, \end{aligned}$$

since $f_X(x) = 0.25$ for all x (Uniform distribution with 'base length' 4, hence 'height' must be 0.25).

For $Y2$:

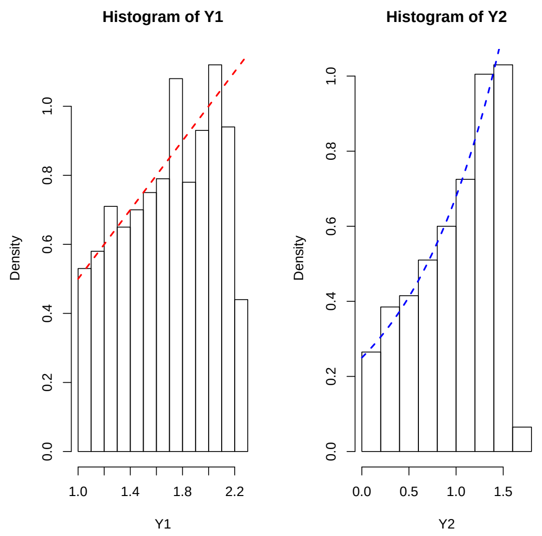
$$F_{Y2}(y) = \Pr(Y_2 \leq y) = \Pr(\log X \leq y) = \Pr(X \leq e^y) = F_X(e^y).$$

Now

$$\begin{aligned} f_{Y2}(y) &= \frac{d}{dy} F_X(e^y) \\ &= f_X(e^y) \times e^y \quad \text{by chain rule} \\ &= \frac{e^y}{4}, \quad \log 1 \leq y \leq \log 5. \end{aligned}$$

(c) Code:

```
par(mfrow=c(1, 2))
hist(Y1, probability = TRUE)
curve(0.5*x, from=1, to=2.3, add=TRUE, col="red", lwd=2, lty=2)
hist(Y2, probability=TRUE)
curve(0.25*exp(x), from=0, to=1.6, add=TRUE, col="blue", lwd=2, lty
    =2)
```



2. (a) Code:

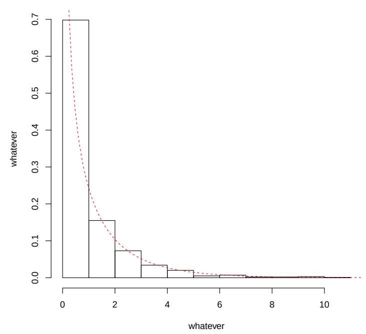
```
Y = (rnorm(1000))^2
```

(b) Code:

```
hist(Y, probability=TRUE, main="", xlab="whatever", ylab="whatever")
```

(c) Code:

```
curve((1/(2^(0.5)*gamma(0.5)))*x^(-0.5)*exp(-x/2), from=0, to=12, col
    ="red", lty=2, add=TRUE)
```



(d) Code:

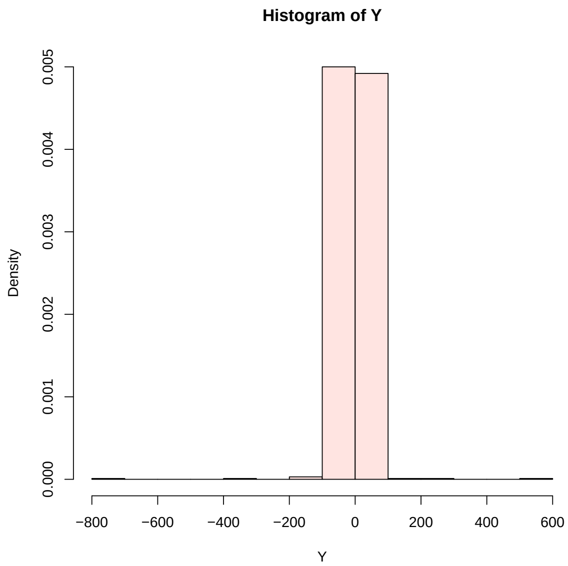
```
length(Y[Y>3.84])/length(Y)
[1] 0.041
```

(e) Code:

```
Y1 = rchisq(1000,1)
1-pchisq(3.84,1)
[1] 0.05004352
```

3. Code:

```
Y = tan(runif(1000, -p1/2, pi/2))
hist(Y, probability=TRUE, col="misty rose")
```



Looks very odd - with *very* heavy tails! What is it??

We can use the transformation method, in exactly the same way we have in previous questions, to find that

$$f_Y(y) = \frac{1}{\pi(1+y^2)}, \quad -\infty < y < \infty.$$

This is the PDF of the Cauchy distribution:

```
Y2 = rcauchy(1000)
```