Bivariate distributions

1. Suppose that $X \sim N(100, 25)$ and that

$$f_{Y|X}(Y|X=x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(y-x)^2}.$$

Write an R function to sample realizations from (X, Y), and explore the dependence between X and Y. [Hint: Do you recognize the distribution of $Y|X$? See Examples 4.1 and 4.2 in the lecture notes for more help]

2. Suppose

$$\begin{pmatrix} X \\ Y \end{pmatrix} \sim N_2\left(\begin{pmatrix} 1 \\ 3 \end{pmatrix}, \begin{pmatrix} 2 & -0.8\sqrt{2} \\ -0.8\sqrt{2} & 1 \end{pmatrix} \right).$$

   (a) Write down the values of μ_X, μ_Y, σ_X and σ_Y.

   (b) Find the correlation ρ.

   (c) Find the conditional distribution for $Y|X=x$, and use this to simulate a sample of 1000 from the bivariate distibution for $(X, Y)^T$. Produce a plot of X versus Y, and use this to confirm your answer to part (b). [Hint: See result 2 on page 25 of the lecture notes, and Example 4.3]

   (d) We will now repeat the simulation exercise in part (c) using the `MASS` package. Set up your mean vector and variance matrix with the following code:

```
mu=c(1,3)
sigma=matrix(data=c(2,-0.8*sqrt(2),-0.8*sqrt(2),1),nrow=2,byrow=TRUE)
```

   Now install the `MASS` package, which can be used to simulate observations from a multivariate normal distribution; then explore the `mvrnorm` function:

```
library(MASS)
?mvrnorm
```

   Simulate a sample of 1000 pairs from the bivariate distribution for $(X, Y)^T$ using `mvrnorm`, and produce a plot of X versus Y. [Hint: See Example 4.4 in the lecture notes]