1. (a) Start with the result:

$$\int xe^{cx^2}dx = \frac{1}{2c}e^{cx^2}.$$

Check:

$$\frac{d}{dx}\left\{\frac{1}{2c}e^{cx^2}\right\} = \frac{1}{2c}\frac{d}{dx}e^{cx^2}.$$

Let $y = e^{cx^2} = e^t$, where $t = cx^2$. Then

$$\frac{dy}{dt} = e^t, \quad \frac{dt}{dx} = 2cx.$$

Therefore

$$\frac{dy}{dx} = e^t \times 2cx = 2cxe^{cx^2}.$$

Thus

$$\frac{d}{dx}\left\{\frac{1}{2c}e^{cx^2}\right\} = \frac{1}{2c} \times 2cxe^{cx^2} = xe^{cx^2}.$$

For the Rayleigh distribution, we have $c = -1/2$. Therefore

$$\begin{aligned} F_X(x) &= \int_0^x te^{-t^2/2} \\ &= \left[\frac{1}{2\times -1/2}e^{-t^2/2}\right]_0^x \\ &= \left[-e^{-t^2/2}\right]_0^x \\ &= \left(-e^{-x^2/2}\right) - (-e^0) \\ &= 1 - e^{-x^2/2}, \quad x > 0. \end{aligned}$$

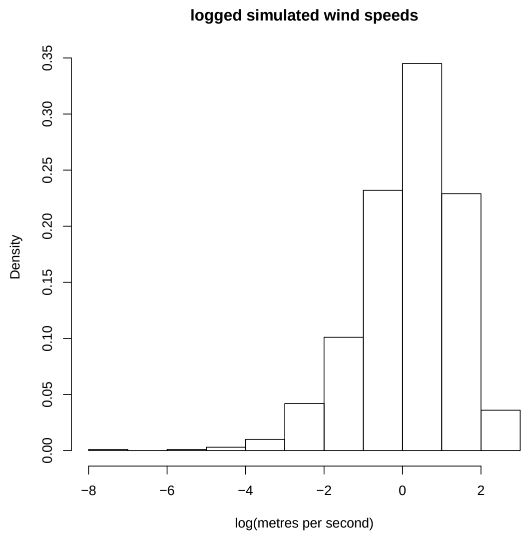
So, option **B**.

(b) Recall from practical 2 that we can simulate `n` realisations from a chi-squared distribution on `v` degrees of freedom using `rchisq(n, v)`; i.e., for 1000 observations from a chi-squared distribution on 2 degrees of freedom:

```
X = rchisq(1000, 2)
```

(i) Then:

```
Y = log(X)
hist(Y)
```



(ii) For the interquartile range:

```
summary(Y)
    Min.  1st Qu.   Median     Mean  3rd Qu.     Max.
-7.0338  -0.4843   0.3247   0.1781   1.0418   2.8636
```

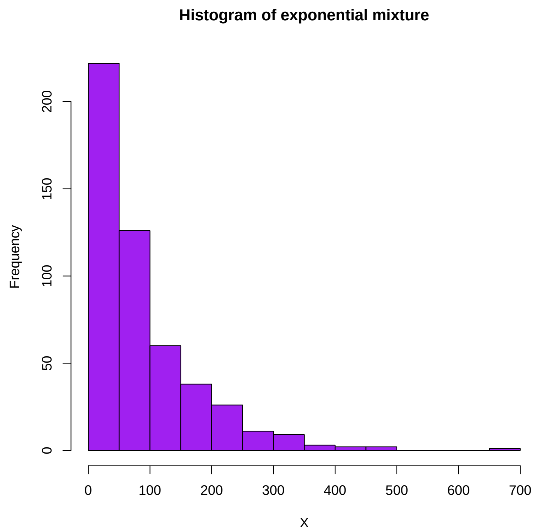
Giving $IQR = 1.0418 - (-0.4843) = 1.5261$.

2. (a) Code for exponential mixture:

```
expmix = function(n, lambda1, w1, lambda2, w2){
  p = c(w1, w2)
  x = vector(mode = "numeric", length = n)
  for(i in 1:n){
    j = sample(c(1,2), 1, prob = p)
    if(j==1){
      x[i]=rexp(1, lambda1)}
    else{
      x[i]=rexp(1, lambda2)}
      }
  return(x)
  }
```

(b) Now:

```
X = expmix(500, 0.01, 0.95, 2, 0.05)
hist(X, col="purple", xlab="X", main="Histogram of exponential
    mixture")
```

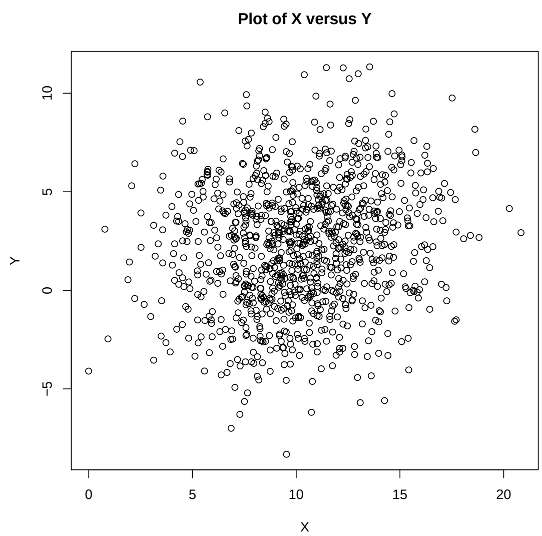


3. (a) Code:

```
Q3 = function(n){
  x = rnorm(n, 10, sqrt(10))
  y = rnorm(n, log(abs(x)), sqrt(10))
  output = matrix(0, 2, n)
  output[1, ] = x
  output[2, ] = y
  return(output)}
```

(b) Now putting the code to use:

```
test = Q3(1000)
plot(test[1,], test[2,], xlab="X", ylab="Y", main = "Plot of X versus
    Y")
```



(c) Correlation:

```
cor(test[1,],test[2,])
[1] 0.1435927
```