Assignment

This assignment is worth 10% of the MAS2602 module marks. Solutions should be handed in to the General Office n the usual way by **3pm on Thursday 21st November**. For information about the late work policy, see Blackboard or the preliminary module information given in the module notes.

Your work should be typed up and any necessary graphs or code included in your typed document. Marks will be awarded for style and presentation.

Question 1 [Total 25 marks]

In this question, you are required to use Monte Carlo Integration to approximate $\int_0^1 f(x)dx$, where

$$f(x) = abx^{a-1}(1 - x^a)^{b-1}$$

for $0 \leq x \leq 1$ and where $a > 0$ and $b > 0$.

(a) Each student will need to generate their own unique values for a and b. To generate *your* values for a and b, type the following code into R:

```
set.seed(LOGIN_ID)
a = sample(3:14, 1); b = sample(3:14, 1)
```

where LOGIN_ID is your computer ID without the 'a' or 'b' at the start. In your solutions, write down your values for a and b and the resulting integrand. For example, if your generated values for a and b were 10 and 12 respectively, you would write down:

$$a = 10; \quad b = 12; \quad f(x) = 120x^9(1 - x^{10})^{11}.$$

(b) Use R to produce a plot of $f(x)$. Within the `plot` command, use the argument `xlim = c(-1, 2)`. Use the `abline` command to indicate a suitable *simulation grid* on your plot. Include this plot in your solutions, with appropriately labelled axes and title. **[7 marks]**

(c) Using a `for` loop over $N = 10^6$ replications, estimate your integral using Monte Carlo Integration. Include your R code in your solutions, which should be nicely formatted/indented. Where necessary, include comments in your code so it is clear what you are trying to do. In your solutions, make it obvious what your estimated integral is. **[18 marks]**

Turn over for questions 2 and 3

Question 2 [Total 25 marks]

Suppose that X is a random variable with PDF

$$f_X(x) = \frac{x}{\sigma^2} e^{-x^2/(2\sigma^2)} \quad \text{for } x > 0$$

where σ is a fixed constant.

(a) Find the CDF $F_X(x)$ of X. Make sure you include some working, in your solutions. **[4 marks]**

(b) Write a function to generate simulated values of X using the inverse CDF method. Include the R code for this function in your solutions. **[8 marks]**

(c) Let d be the last digit of your student ID. Let $\sigma = 1 + (d/2.5)$. Write down your value of d, and plot a density histogram of a sample of X of size 10,000 using your value of σ. Include this histogram in your solutions; again, make sure axes are appropriately labelled. **[5 marks]**

(d) Check that your function has worked by superimposing a curve on your histogram showing the PDF. **[2 marks]**

(e) The distribution you have simulated from is often used in the physical sciences to model wind speeds and wave heights; it is also used to model noise variation in magnetic resonance imaging (MRI). Can you find the name of this distribution? **[2 marks]**

(f) Use your simulated valued of X to verify the expectation and variance of the distribution you identified in part (e). **[4 marks]**

Question 3 [Total 25 marks]

(a) Write an R function to sample from the log-normal distribution $LN(\mu, \sigma^2)$ (see section 3.1 of the notes and example 3.3). Make sure you function takes three input arguments: the number of realisations to generate (`N`), the mean (`mu`) and the standard deviation (`sigma`). Include your R code in your solutions. **[5 marks]**

(b) For $\sigma = 2$ generate samples from the log-normal distribution for a number of different values for μ in the range $\mu \in (0, 5)$. For each value of μ compute the log of the sample mean. Plot a graph of the log of the sample mean versus μ, and include this plot in your solutions. **[8 marks]**

(c) Repeat part (b), plotting the log of the sample median versus μ. **[8 marks]**

(d) What can you conclude about the mean and median of the $LN(\mu, \sigma^2)$ distribution? **[4 marks]**