Learning outcomes: Chapter 4

1. You should understand the difference between a *point estimate* and an *interval estimate*.

2. You should be able to define a *frequentist confidence interval*, a *Bayesian confidence interval* and a *highest density interval*, and you should understand the differences in interpretation between these intervals.

3. You should be able to determine a highest density interval for θ in the following simple situations:

- When the posterior for θ is a beta distribution and the lower/upper bounds of the interval are clearly 0/1;
- When the posterior for θ is symmetric, e.g. a Normal distribution.

4. You should be able to write simple functions in R to obtain a highest density interval for any unimodal posterior distribution.

5. You should memorise, and be able to use, the following result to obtain the predictive probability density function for a future observation y:

$$f(y|\boldsymbol{x}) = \int_{\Theta} f(y|\theta)\,\pi(\theta|\boldsymbol{x})\,d\theta.$$

6. You should be able to obtain prediction intervals in simple examples for discrete θ, or when the predictive distribution is of known form.

7. You should be able to state, and prove, the following result for the predictive distribution for Y when $\boldsymbol{X}$ and Y are independent:

$$f(y|\boldsymbol{x}) = \frac{f(y|\theta)\pi(\theta|\boldsymbol{x})}{\pi(\theta|\boldsymbol{x}, y)}.$$