

Learning outcomes: Chapter 3

1. You should know how to obtain parameters for a prior distribution for θ given some suggested prior summaries for θ . For example,
 - parameters for the gamma distribution for θ given $E(\theta)$ and $Var(\theta)$;
 - parameters for the beta distribution for θ given the mode for θ and a probability for θ .
2. You should be able to describe the *trial roulette* and *bisection* methods of prior elicitation. You should also be able to:
 - Interpret output from the *MATCH* elicitation tool;
 - explain the processes of *feedback* and *refinement*;
 - explain the role of *feedback percentiles*, and be able to transpose these into suitable questions to ask a non-statistician.
3. You should be able to define *substantial prior information*.
4. You should be able to explain how we might operate Bayes' theorem with *vague prior knowledge*, and you should be able to obtain posterior distributions in circumstances when our prior knowledge about θ is indeed vague.
5. You should be able to explain *prior ignorance*; you should also memorise the form of Jeffreys' prior for θ :

$$\pi(\theta) \propto \sqrt{I(\theta)}$$

where

$$I(\theta) = E_{\mathbf{X}|\theta} \left[-\frac{\partial^2}{\partial \theta^2} \log f(\mathbf{X}|\theta) \right],$$

and be able to obtain this prior for a given model $f(\mathbf{x}|\theta)$.

6. You should be able to state, and provide an outline proof of, the asymptotic posterior distribution for θ :

$$\theta|\mathbf{x} \sim N\left(\hat{\theta}, J(\hat{\theta})^{-1}\right) \quad \text{approximately,}$$

where $J(\theta)$ is the *observed information*

$$J(\theta) = -\frac{\partial^2}{\partial \theta^2} \log f(\mathbf{X}|\theta).$$

7. Given a model $f(\mathbf{x}|\theta)$, you should be able to obtain the asymptotic posterior distribution for θ .