Extra Workshop on Probability/Probability Distributions

Wednesday 3rd December
2pm – 4pm
Herschel Building, Lecture Theatre 1

Question 1 (a)

Weekly sales at H&M over the past year are summarised in the following histogram. Over the past year weekly sales have averaged at about £60,000 with a standard deviation of £12500. Let X be weekly sales (in £).

Distribution: $X \sim N\left(\mu, \sigma^2\right)$
$X \sim N\left(60000, 12500^2\right)$

Question 1 (b)

The probability of receiving a bonus at a particular company is 0.05. The Newcastle branch of this company employs eighteen people. Let Y be the number of people who receive a bonus at this branch.

Distribution: $Y \sim Bin\,(n, p)$

$$Y \sim Bin\,(18, 0.05)$$

Question 1 (c)

A parachutist jumps from a plane and will land randomly somewhere between village A and village B. Villages A and B are two miles apart. Let X be the ditance (in miles) the parachutist lands from town A.

Distribution: $X \sim U(a, b)$

$$X \sim U(0, 2)$$

Question 1 (d)

Let Y be the number of people arriving at a queue in a bank in five minute intervals. It is known that, typically, about three people will arrive every five minutes.

Distribution: $Y \sim Po\,(\lambda)$

$Y \sim Po\,(3)$

Question 1 (e)

Let T be the time between arrivals at the bank queue in part (d).

Distribution: $T \sim exp(\lambda)$
$T \sim exp(3)$

Question 2(a)(i)

$$X \sim N\left(60000, 12500^2\right)$$

$$\begin{aligned} P(X < 45000) &= P\left(Z < \frac{X-\mu}{\sigma}\right) \\ &= P\left(Z < \frac{45000-60000}{12500}\right) \\ &= P(Z < -1.2) \\ &= 0.1151 \end{aligned}$$

Question 2(a)(ii)

$$X \sim N\left(60000, 12500^2\right)$$

$$\begin{aligned} P(X > 92000) &= 1 - P(X < 92000) \\ &= 1 - P\left(Z < \frac{92000 - 60000}{12500}\right) \\ &= 1 - P(Z < 2.56) \\ &= 1 - 0.9948 \\ &= 0.0052. \end{aligned}$$

Question 2(a) (iii)

$$X \sim N\left(60000, 12500^2\right)$$

$$\begin{aligned}
P(50000 < X < 75000) &= P(X < 75000) - P(X < 50000) \\
&= P\left(Z < \frac{75000 - 60000}{12500}\right) \\
&\quad -P\left(Z < \frac{50000 - 60000}{12500}\right) \\
&= P(Z < 1.2) - P(Z < -0.8) \\
&= 0.8849 - 0.2119 \\
&= 0.673.
\end{aligned}$$

Question 2(b)(i)

$$Y \sim Bin\,(18, 0.05)$$

$$\begin{aligned} E[Y] &= n \times p \\ &= 18 \times 0.05 \\ &= 0.9 \end{aligned}$$

Question 2(b)(ii)

$$Y \sim Bin(18, 0.05)$$

$$\begin{aligned} Var(Y) &= n \times p \times (1-p) \\ &= 18 \times 0.05 \times 0.95 \\ &= 0.855 \end{aligned}$$

Thus, the standard deviation is given by

$$\begin{aligned} s.d.(Y) &= \sqrt{0.855} \\ &= 0.925 \end{aligned}$$

Question 2(b)(iii)

$$Y \sim Bin\,(18, 0.05)$$

$$\begin{aligned} P(Y = 3) &= {}^{n}C_{r} \times p^{r} \times (1 - p)^{n-r} \\ &= {}^{18}C_{3} \times 0.05^{3} \times 0.95^{15} \\ &= 816 \times 0.000125 \times 0.463 \\ &= 0.0473 \end{aligned}$$

Question 2(b)(iv)

$$Y \sim Bin\,(18, 0.05)$$

$$\begin{aligned}
P(Y \leq 2) &= P(Y = 0) + P(Y = 1) + P(Y = 2) \\
&= {}^{18}\text{C}_0 \times 0.05^0 \times 0.95^{18} \\
&\quad + {}^{18}\text{C}_1 \times 0.05^1 \times 0.95^{17} \\
&\quad + {}^{18}\text{C}_2 \times 0.052 \times 0.95^{16} \\
&= 0.397 + 0.376 + 0.168 \\
&= 0.941.
\end{aligned}$$

Question 2(c)(ii)

$$X \sim U(0, 2)$$

$$\begin{aligned} E[X] &= \frac{a+b}{2} \\ &= \frac{0+2}{2} \\ &= 1 \end{aligned}$$

Question 2(c)(iii)

$$X \sim U\,(0, 2)$$

$$\begin{aligned} Var(X) &= \frac{(b-a)^2}{12} \\ &= \frac{(2-0)^2}{12} \\ &= \frac{4}{12} \\ &= 0.333 \end{aligned}$$

Thus, we have

$$\begin{aligned} s.d.(X) &= \sqrt{0.333} \\ &= 0.577 \end{aligned}$$

Question 2(c)(iv)

$$X \sim U(0, 2)$$

$$\begin{aligned} P(X < 0.5) &= \frac{x - a}{b - a} \\ &= \frac{0.5 - 0}{2 - 0} \\ &= 0.25 \end{aligned}$$

Question 2(c)(v)

$$X \sim U(0, 2)$$

$$\begin{aligned}
P(X > 1) &= 1 - P(X < 1) \\
&= 1 - \frac{x - a}{b - a} \\
&= 1 - \frac{1 - 0}{2 - 0} \\
&= 1 - 0.5 \\
&= 0.5.
\end{aligned}$$

Question 2(d)(i), 2(d)(ii)

$$Y \sim Po(3)$$

$$\begin{aligned} E(Y) &= \lambda \\ &= 3 \end{aligned}$$

$$\begin{aligned} Var(Y) &= \lambda \\ &= 3 \qquad \text{and so} \end{aligned}$$

$$\begin{aligned} s.d.(Y) &= \sqrt{3} \\ &= 1.732. \end{aligned}$$

Question 2(d)(iii)

$$Y \sim Po(3)$$

$$\begin{aligned} P(Y = 5) &= \frac{e^{-\lambda} \times \lambda^r}{r!} \\ &= \frac{e^{-3} \times 3^5}{5!} \\ &= 0.1008 \end{aligned}$$

Question 2(d)(iv)

$$Y \sim Po(3)$$

$$\begin{aligned}
P(Y \geq 3) &= P(Y = 3) + P(Y = 4) + P(Y = 5) + \ldots \\
&= 1 - \{P(Y = 0) + P(Y = 1) + P(Y = 2)\} \\
&= 1 - \{0.0498 + 0.1494 + 0.224\} \\
&= 0.5768.
\end{aligned}$$

Question 2(e)(i)

$$T \sim exp(3)$$

$$\begin{aligned} E[T] &= \frac{1}{\lambda} \\ &= \frac{1}{3} \\ &= 0.333 \end{aligned}$$

$$\begin{aligned} Var(T) &= \frac{1}{\lambda^2} \\ &= \frac{1}{9} \\ &= 0.111 \end{aligned}$$

Question 2(e)(ii)

$$T \sim exp(3)$$

$$\begin{aligned} P(T < 0.5) &= 1 - e^{-\lambda \times t} \\ &= 1 - e^{-3 \times 0.5} \\ &= 1 - 0.223 \\ &= 0.777. \end{aligned}$$