Where we are in the course

Weeks 1–4 (Data collection and summaries)

- How to *collect* data
- How to *summarise* data
  - Tabular
  - Graphical
  - Numerical (location and spread)

Where we are in the course

Weeks 5–7 (Probability)

- Introduction to probability
  - Interpretations of probability
  - Laws of probability
- Conditional probability and probability trees
- *EMV* and decision trees

Don't forget:

- Computer practical this week in place of tutorial!
- CBA2 deadline midnight this Friday!

Where we are in the course

Weeks 8–11 (Probability models)

- Models for discrete data
  - The Binomial distribution
  - The Poisson distribution
- Models for continuous data
  - The Normal distribution
  - The Uniform distribution
  - The exponential distribution

Lecture 7

DECISION MAKING USING PROBABILITY

Decision–making using probability

In this lecture, we look at how we can use probability in order to aid management **decision–making**.

Expected Monetary Value

Intuition should now help to explain how probability can be used to aid the decision–making process.

For example, suppose we're considering launching a new product on the market. We conduct a pre–launch questionnaire and 86 out of the 100 questionnaire respondents say that they *would* buy our product if it was on the market. Thus,

$$P(\text{product successful}) \;=\; \frac{86}{100} = 0.86,$$

which is quite good, and so surely we should launch the product? It looks promising!

Expected Monetary Value

But... we should also consider the **financial outcome** of our situation.

For example, if the product is successful, we might make **thousands**, but if the product is *not* successful, we could stand to lose **millions**!

Such financial considerations could outweigh the high probability of success alone.

Expected Monetary Value

So, in real–life scenarios, not only do we use **probability** to aid the decision–making process, but also the **financial implications**.

This is achieved by weighting the probability of different outcomes by their value, which is often financial.

The **Expected Monetary Value** (EMV) of a single event is simply the probability of that event multiplied by the monetary value of that outcome.

Example 1

For example, if you win £5 if you pulled an ace from a pack of cards, the *EMV* would be

$$EMV(Ace) = \frac{4}{52} \times 5 = 0.38.$$

In other words, if you repeated this bet a large number of times, overall you would come out, on average, 38 pence better off per bet.

Example 2

Consider another bet. When rolling a die,

- if it's a six you have to *pay* £5
- if it's any other number you *receive* £2.50

Would you take on this bet?

Example 2

Probability	**Financial outcome**
$P(6) = 1/6$	$-£5$
$P(\text{Not a } 6) = 5/6$	$£2.50$

Therefore

$$
\begin{aligned}
EMV(\text{Six}) &= \frac{1}{6} \times -5.00 = -0.833 \\
EMV(\text{Not a Six}) &= \frac{5}{6} \times 2.50 = 2.0833
\end{aligned}
$$

and hence the expected monetary value of the bet is

$$EMV(\text{Bet}) = -0.833 + 2.083 = 1.25.$$

Therefore, in the long run, this would be a bet to take on as it has a **positive expected monetary value**.

Example 3: The National Lottery

In a recent lottery draw, the prizes were

Number of balls matched	Probability	Prize
6	0.000000071	£2.4M
5 plus bonus	0.000000429	£240K
5	0.000018449	£3K
4	0.000968619	£100
3	0.0177	£10
< 3	0	£0

Example 3: The National Lottery

The *EMV* of the bet is

$$
\begin{aligned}
EMV &= P(\text{match 6 balls}) \times \text{Prize(match 6 balls)} \\
&\quad + P(\text{5 plus bonus}) \times \text{Prize(5 plus bonus)} \\
&\quad + P(\text{match 5 balls}) \times \text{Prize(match 5 balls)} \\
&\quad + P(\text{match 4 balls}) \times \text{Prize(match 4)} \\
&\quad + P(\text{match 3 balls}) \times \text{Prize(match 3 balls)} \\
\\
&= 2.4M \times 0.000000071 + \ldots + 10 \times 0.0177 \\
&= \pounds 0.6176.
\end{aligned}
$$

Therefore, a fair price for a ticket in this particular lottery is around 62p. The difference between this and the standard £1 charge for a ticket goes to "good causes" and, of course, **Camelot's profits**.

In general, the expected monetary value of a project or bet is given by the formula

$$EMV = \sum P(\text{Event}) \times \text{ Monetary value of Event}$$

where the sum is over all possible events.

The *EMV* of a project can be used as a **decision criterion** for choosing between different projects and has applications in a large number of situations.

Example 4

A small company is trying to decide how to launch a new and innovative product.

It could go for a direct approach, launching onto the whole of the domestic market through traditional distribution channels, or it could launch only on the internet.

A third option exists where the product is licensed to a larger company through the payment of a licence fee irrespective of the success of the product.

How should the company launch the product?

Example 4

The company has done some initial market research and the managing director believes the probability of the product being successful can be classed into three categories:

High **Medium** **Low**

She thinks that these categories will occur with probabilities 0.2, 0.35 and 0.45 respectively and her thoughts on the likely profits (in £K) to be earned in each plan are

	High	Medium	Low
Direct	100	55	-25
Internet	46	25	15
Licence	20	20	20

The *EMV* of each plan can be calculated as follows:

$$\begin{aligned} EMV(\textbf{Direct}) &= 0.2 \times 100 + 0.35 \times 55 + 0.45 \times (-25) = \text{£}28\text{K} \\ EMV(\textbf{Internet}) &= 0.2 \times 46 + 0.35 \times 25 + 0.45 \times 15 = \text{£}24.7\text{K} \\ EMV(\textbf{Licence}) &= 0.2 \times 20 + 0.35 \times 20 + 0.45 \times 20 = \text{£}20\text{K}. \end{aligned}$$

On the basis of expected monetary value, the best choice is the Direct approach.

Decision trees

In the last example we had to make a **decision**.

When we include a decision in a tree diagram (see last week) we use a rectangular node, called a **decision node** to represent the decision.

The diagram is then called a **decision tree**.

There are no probabilities at a decision node but we evaluate the expected monetary values of the options.

In a decision tree the first node is always a decision node. There may also be other decision nodes. If there is another decision node then we evaluate the options there and choose the best and the expected value of this option becomes the expected value of the branch leading to the decision node.

Example 4 revisited

A small company is trying to decide how to launch a new and innovative product.

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Example 4 revisited

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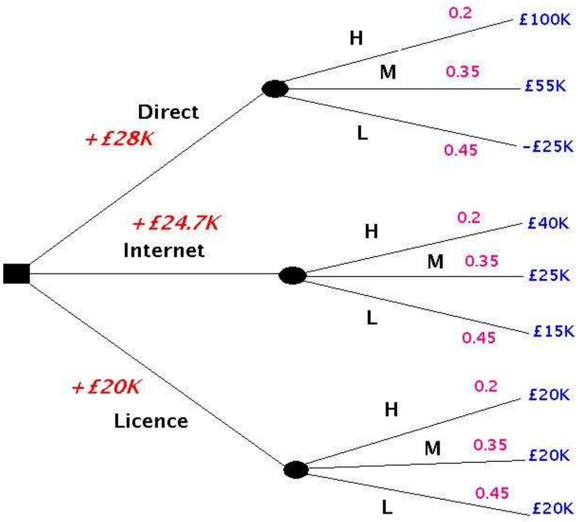
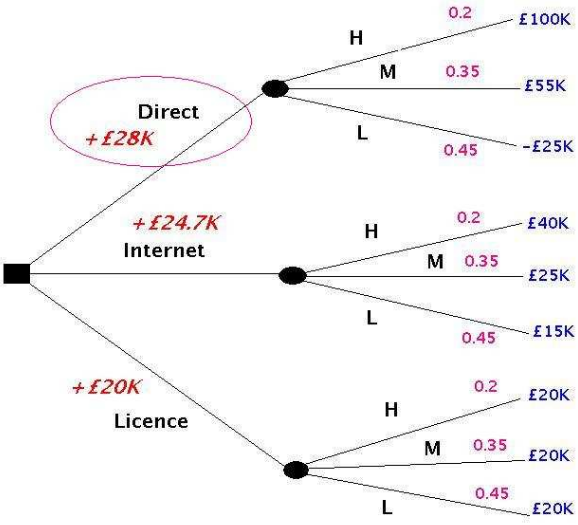
Example 4 revisited



Example 4 revisited



A more complicated example

The manager of a small I.T. sales company has the opportunity to buy a fixed quantity of a new type of soundcard for home PCs which they can then offer for sale to clients.

The decision to buy the product and offer it for sale would involve a fixed cost of £200,000. The number of soundcards that would be sold is uncertain, but the manager's prior beliefs are expressed as follows.

- Sales will be "poor" with probability 0.2; this will result in an income of £100,000.
- Sales will be "moderate" with probability 0.5; this will result in an income of £220,000.
- Sales will be "good" with probability 0.3; this will result in an income of £350,000.

A more complicated example

For an additional fixed cost of £30,000, market research can be conducted to aid the decision–making process.

The outcome of the market research can be either positive or negative, with probabilities 0.58 and 0.42 respectively. Knowing the outcome of the market research changes the probabilities for the main sales project as follows:

	Main sales probabilities		
Market research	Poor	Moderate	Good
Positive	0.15	0.45	0.4
Negative	0.6	0.35	0.05

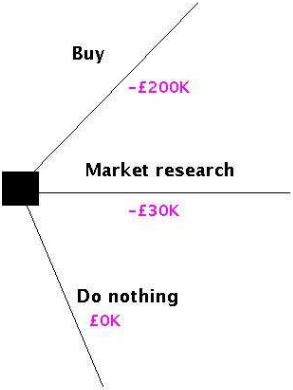
A more complicated example

The manager will make decisions based on expected monetary value.

(a) Draw a decision tree for this problem.

(b) Use expected monetary value to determine the optimal course of action for the company.

A more complicated example



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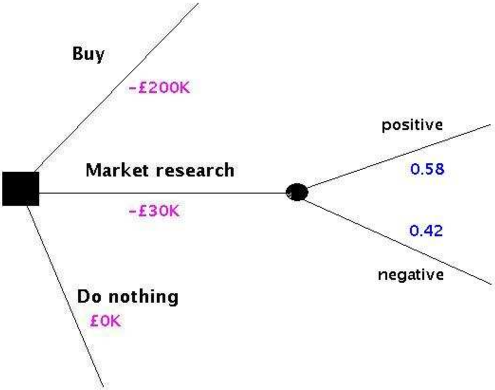

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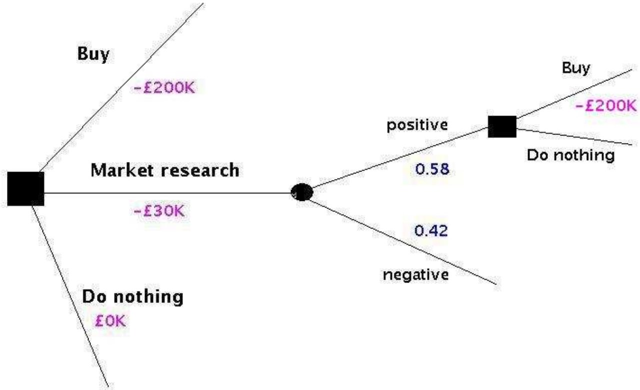

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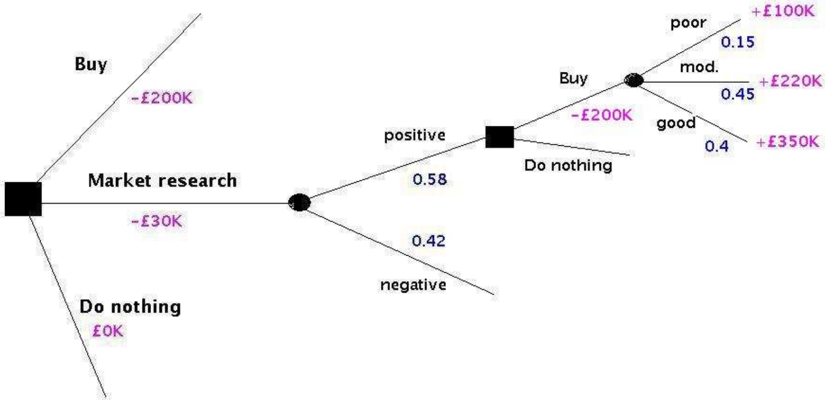

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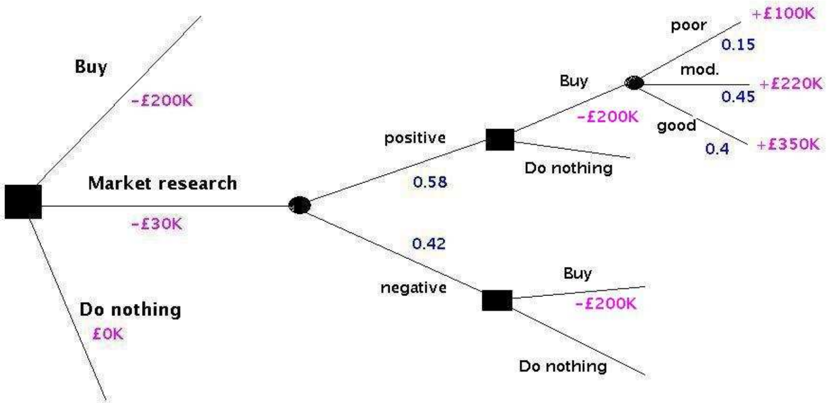

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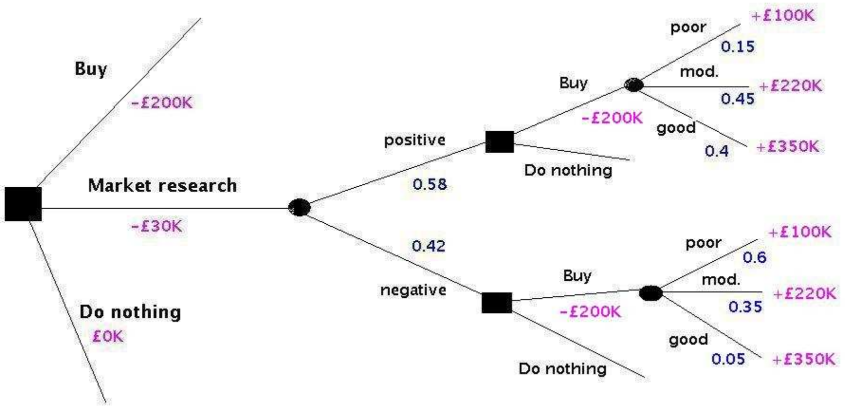

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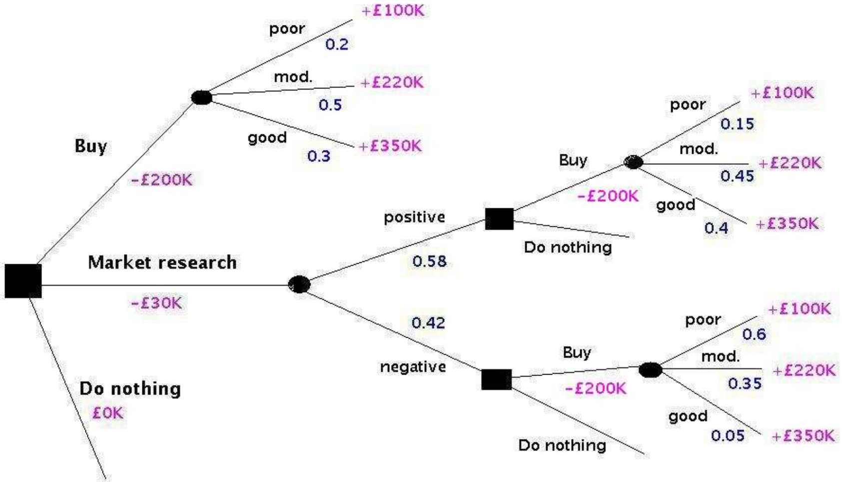

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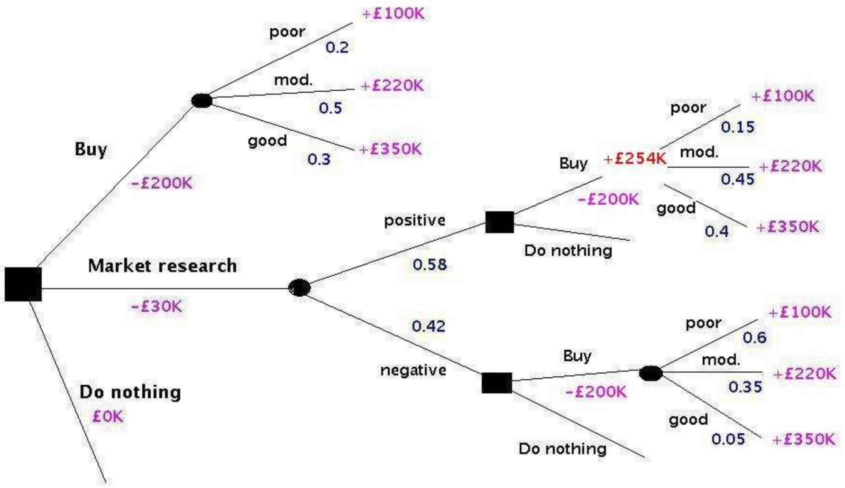

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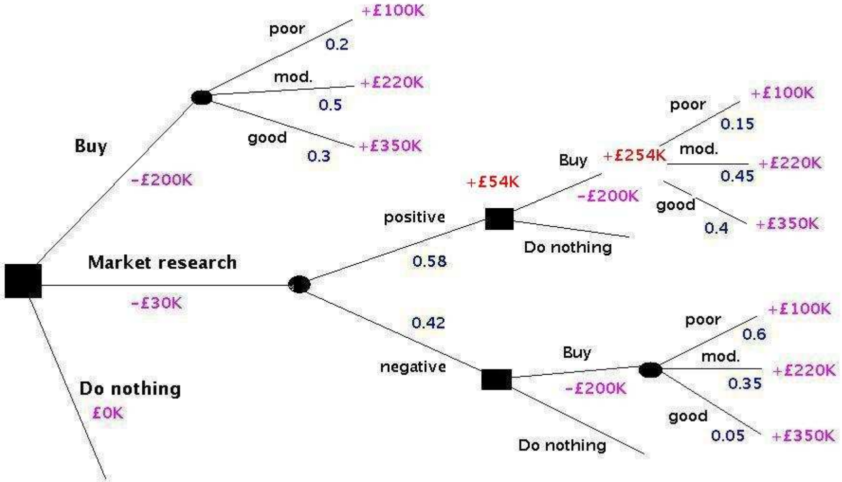

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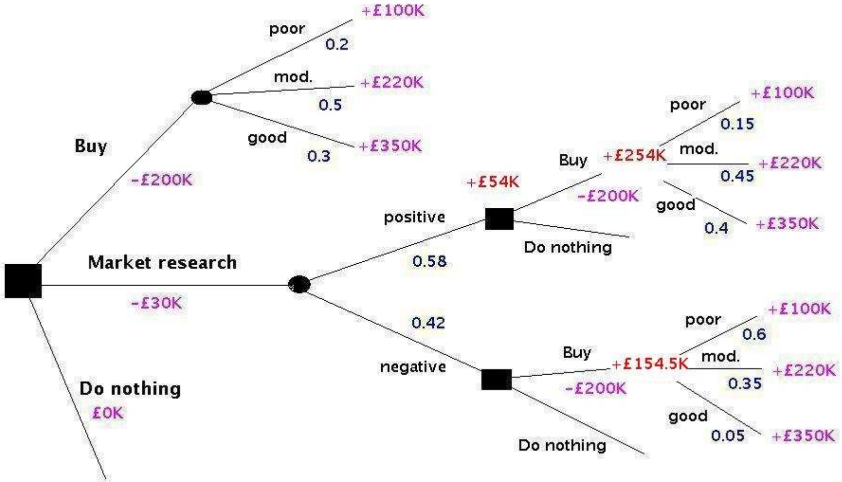

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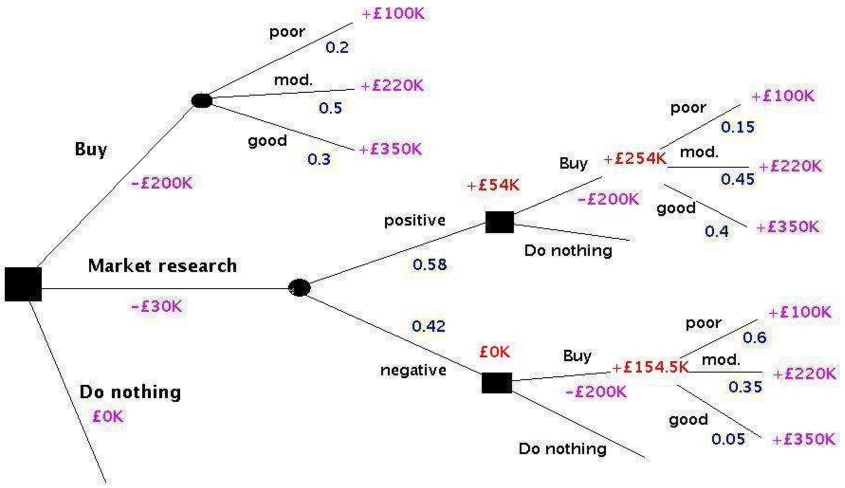

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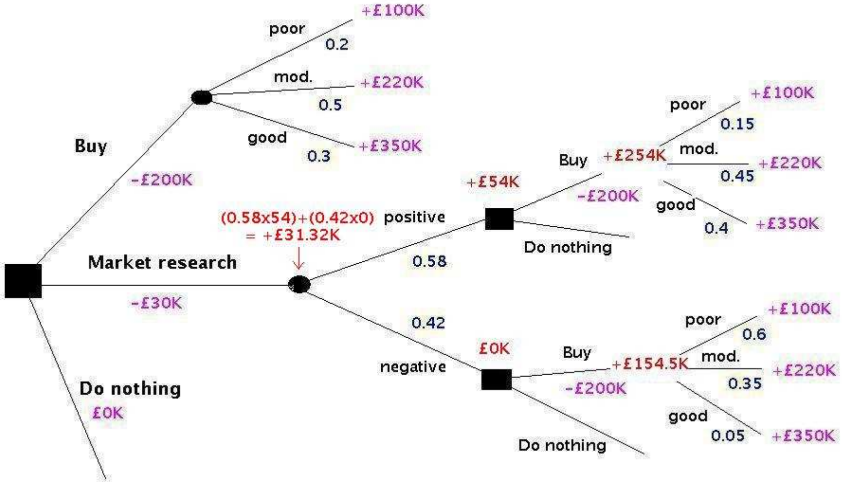

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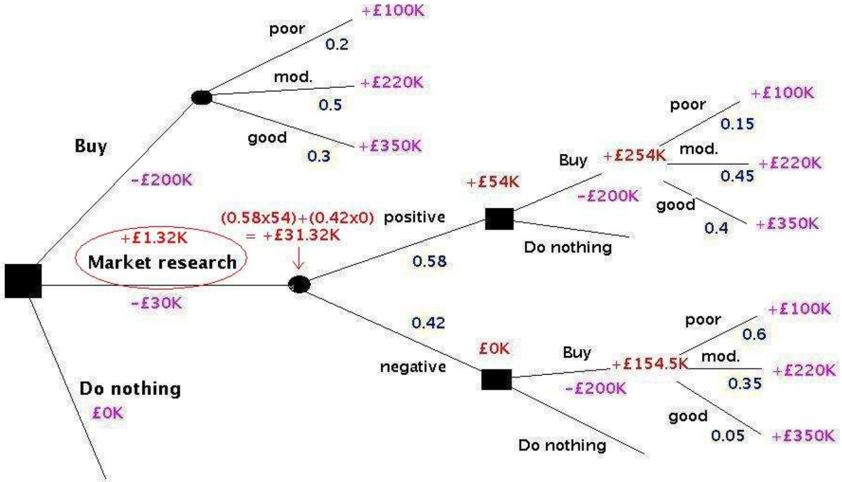

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