Solutions to Exercises 8

1. The number of ways of drawing 6 balls from 48 balls is

$$\begin{aligned}
{}^{48}\mathrm{C}_6 &= \frac{48!}{6! \times (48-6)!} \\
&= \frac{48!}{6! \times 42!} \\
&= \frac{48 \times 47 \times 46 \times 45 \times 44 \times 43}{6 \times 5 \times 4 \times 3 \times 2 \times 1} \\
&= 12,271,512.
\end{aligned}$$

As there is only one selection that matches the 6 balls drawn, the probability of winning the jackpot in this lottery is

$$\frac{1}{12,271,512} = 0.00000008149.$$

2. This is a question about permutations as the ordering is important. The number of permutations of 4 features from 10 features is

$${}^{10}\mathrm{P}_4 = \frac{10!}{(10-4)!} = \frac{10!}{6!} = \frac{10 \times 9 \times 8 \times 7 \times 6!}{6!} = 5040.$$

As there is only one ordering that matches my preferred ordering, the probability of choosing my preferred ordering is

$$\frac{1}{5040} = 0.0001984.$$

3. There are 10 choices for the 1st digit, and 10 choices for the second digit, and so on. Therefore the number of possible 7 digit phone numbers is

$$10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 10,000,000$$

and so the probability of randomly selecting my unique telephone number is

$$\frac{1}{10,000,000} = 0.0000001.$$

4. (a) Assuming that calls are answered independently, $X \sim Bin(20, 0.85)$.

(b) The mean and variance are

$$\begin{aligned}
E(X) &= np = 20 \times 0.6 = 12 \\
Var(X) &= np(1-p) = 20 \times 0.6 \times 0.4 = 4.8
\end{aligned}$$

and so

$$SD(X) = \sqrt{Var(X)} = \sqrt{4.8} = 2.19.$$

(c)

$$\begin{aligned} P(X=9) &= {}^{n}\mathrm{C}_{r} p^{r}(1-p)^{n-r} \\ &= {}^{20}\mathrm{C}_{9} \times 0.6^{9} \times (1-0.6)^{11} \\ &= 167960 \times 0.010077696 \times 0.000041943 \\ &= 0.071. \end{aligned}$$

(d)

$$\begin{aligned} P(X<2) &= P(X=0)+P(X=1) \\ &= {}^{20}\mathrm{C}_{0} \times 0.6^{0} \times (1-0.6)^{20-0} + {}^{20}\mathrm{C}_{1} \times 0.6^{1} \times (1-0.6)^{20-1} \\ &= 0.00000001 + 0.000000329 \\ &= 0.000000339. \end{aligned}$$

5. (a) $X \sim Po(10)$

(b) The mean and variance are

$$\begin{aligned} E(X) &= \lambda = 10 \\ Var(X) &= \lambda = 10 \end{aligned}$$

and so

$$SD(X) = \sqrt{Var(X)} = \sqrt{10} = 3.16.$$

(c)

$$\begin{aligned} P(X=12) &= \frac{\lambda^{r} e^{-\lambda}}{r!} \\ &= \frac{10^{12} \times e^{-10}}{12!} \\ &= 0.09478. \end{aligned}$$

(d)

$$\begin{aligned} P(X \leq 2) &= P(X=0)+P(X=1)+P(X=2) \\ &= \frac{10^{0} \times e^{-10}}{0!} + \frac{10^{1} \times e^{-10}}{1!} + \frac{10^{2} \times e^{-10}}{2!} \\ &= e^{-10} + 10e^{-10} + 50e^{-10} \\ &= 0.0000454 + 0.0004540 + 0.0022700 \\ &= 0.00277. \end{aligned}$$