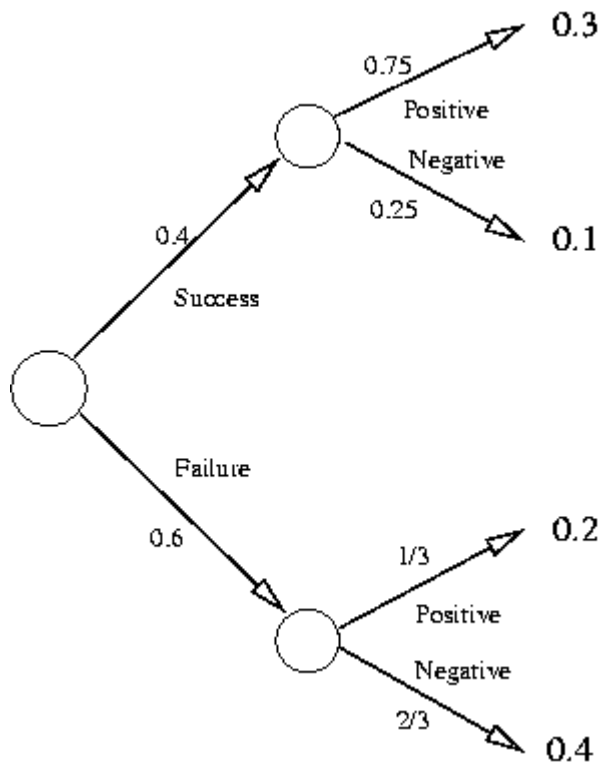


Solutions to Exercises 7

1. Sorry, I've not had time to do full solutions to this question – this question is quite similar to (but easier than!) the question on page 82 of the lecture notes!
2. (a) The probability tree is as follows.



- i. $P(\text{positive}) = 0.3 + 0.2 = 0.5$.
- ii. $P(\text{negative}) = 0.1 + 0.4 = 1 - P(\text{positive}) = 0.5$.
- iii.

$$\begin{aligned}
 P(\text{success} | \text{positive}) &= \frac{P(\text{success and positive})}{P(\text{positive})} \\
 &= \frac{0.3}{0.5} = 0.6
 \end{aligned}$$

iv.

$$\begin{aligned}
 P(\text{failure} | \text{positive}) &= 1 - P(\text{success} | \text{positive}) \\
 &= 1 - 0.6 = 0.4
 \end{aligned}$$

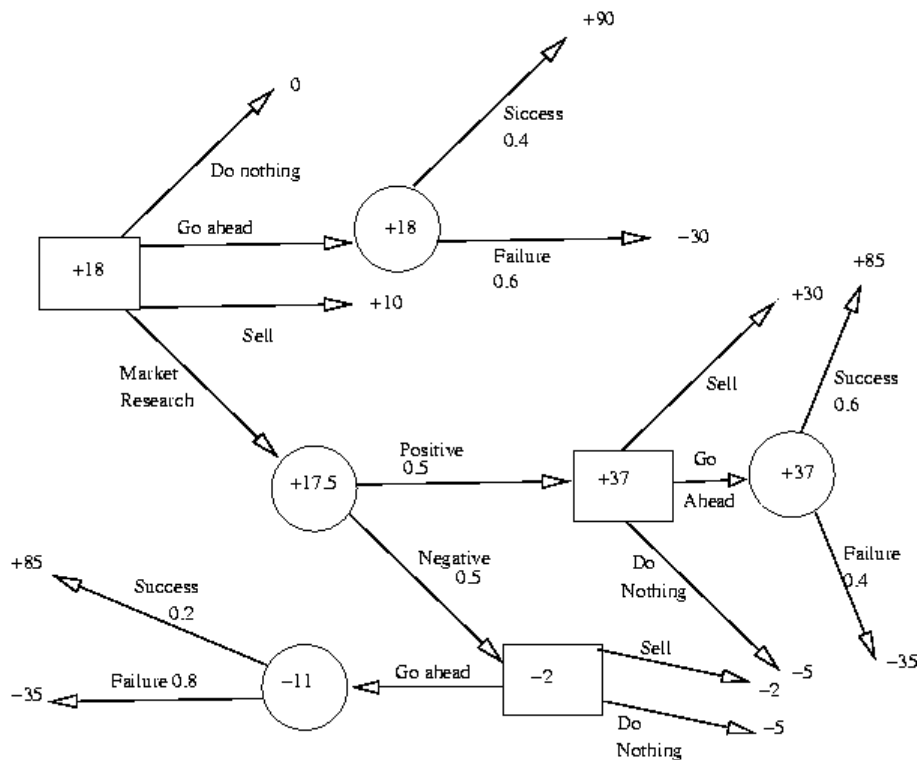
v.

$$\begin{aligned}
 P(\text{success} | \text{negative}) &= \frac{P(\text{success and negative})}{P(\text{negative})} \\
 &= \frac{0.1}{0.5} = 0.2
 \end{aligned}$$

vi.

$$\begin{aligned}
 P(\text{failure} | \text{negative}) &= 1 - P(\text{success} | \text{negative}) \\
 &= 1 - 0.2 = 0.8
 \end{aligned}$$

(b) The tree diagram is as follows.



(c) Market research done, at a cost of £5000. Result positive.

- Sell: $£35,000 - £5,000 = £30,000$.
- Go ahead: Value of success is $£90,000 - £5,000 = £85,000$. Value of failure is $-£30,000 - £5,000 = -£35,000$. The expected value is then

$$0.6 \times £85,000 - 0.4 \times £35,000 = £37,000.$$

- Do nothing more: Value is $-£5,000$.

In these circumstances she should **go ahead** because this gives the greatest expected value.

(d) Market research done, at a cost of £5000. Result negative.

- Sell: $£3,000 - £5,000 = -£2,000$.
- Go ahead: Value of success is $£90,000 - £5,000 = £85,000$. Value of failure is $-£30,000 - £5,000 = -£35,000$. The expected value is then

$$0.2 \times £85,000 - 0.8 \times £35,000 = -£11,000.$$

- Do nothing more: Value is $-£5,000$.

In these circumstances she should **sell** because this gives the greatest expected value (in this case, the smallest expected loss).

(e) The expected value for the option “Market Research” is therefore

$$\begin{aligned} & P(\text{positive}) \times \text{Expected value given positive} + \\ & P(\text{negative}) \times \text{Expected value given negative} \\ = & 0.5 \times £37,000 - 0.5 \times £2,000 = £17,500. \end{aligned}$$

(f) The expected values for the other three options are as follows.

- Do nothing: £0.
- Go ahead without market research:

$$0.4 \times £90,000 - 0.6 \times £30,000 = £18,000.$$

- Sell: £10,000

(g) The owner's best strategy is therefore to **go ahead without market research**. This gives an expected monetary value of £18,000.