Solutions to Exercises 4

1. Let the weight of sack i be x_i. The sum of the weights is

$$\sum_{i=1}^{50} x_i = 505.8.$$

Therefore the sample mean is

$$\bar{x} = \frac{1}{50}\sum_{i}^{50} x_i = \frac{505.8}{50} = 10.116\,\text{kg}.$$

2. Grouping these data into a frequency table gives

Class j	**Class Interval**	**Mid-point** (m_j)	**Frequency** (f_j)
1	$8.0 \leq x < 8.5$	8.25	2
2	$8.5 \leq x < 9.0$	8.75	4
3	$9.0 \leq x < 9.5$	9.25	4
4	$9.5 \leq x < 10.0$	9.75	9
5	$10.0 \leq x < 10.5$	10.25	14
6	$10.5 \leq x < 11.0$	10.75	9
7	$11.0 \leq x < 11.5$	11.25	5
8	$11.5 \leq x < 12.0$	11.75	2
9	$12.0 \leq x < 12.5$	12.25	0
10	$12.5 \leq x < 13.0$	12.75	1
Total (n)			50

3. Using the grouped data, the approximation of the sample mean is

$$\bar{x} \approx \frac{1}{50}\sum_{j=1}^{10} f_j m_j = \frac{1}{50}\left(2 \times 8.25 + 4 \times 8.75 + \cdots + 0 \times 12.25 + 1 \times 12.75\right) = \frac{507}{50} = 10.14\,\text{kg}.$$

This value is fairly close to the correct sample mean (in 1 above).

4. A stem and leaf plot for these data was produced in Exercises 2:

```
Stem-and-leaf of Weight  N  = 50
Leaf Unit = 0.10


  2     8    12
  6     8    5789
 10     9    2334
 19     9    556667799
(14)    10   00000122233444
 17     10   566666789
  8     11   02333
  3     11   56
  1     12
  1     12   8
```

Alternatively, we can put the observations into increasing order.

8.1	8.2	8.5	8.7	8.8	8.9	9.2	9.3	9.3	9.4
9.5	9.5	9.6	9.6	9.6	9.7	9.7	9.9	9.9	10.0
10.0	10.0	10.0	10.0	10.1	10.2	10.2	10.2	10.3	10.3
10.4	10.4	10.4	10.5	10.6	10.6	10.6	10.6	10.6	10.7
10.8	10.9	11.0	11.2	11.3	11.3	11.3	11.5	11.6	12.8

As there are $n = 50$ observations, the median M is the $(50+1)/2 = 25\frac{1}{2}$th smallest observation, that is, half way between the 25th and 26th smallest observations:

$$M = \frac{10.1 + 10.2}{2} = 10.15\,\text{kg}.$$

5. The model class in the grouped frequency table is the class with the largest frequency, that is class 5, with $10.0 \leq x < 10.5$.

6. The range of the data is the difference between the largest and smallest observations. Here the minimum and maximum values are min = 8.1 kg and max = 12.8 kg and so the range is

$$\text{Range } = \text{max } - \text{min } = 12.8 - 8.1 = 4.7\text{kg}.$$

7. The interquartile range is the difference between the upper and lower quartiles. As there are $n = 50$ observations, the lower quartile is the $(50+1)/4 = 12\frac{3}{4}$th smallest observation, that is, three quarters of the way between the 12th and 13th smallest observations:

$$Q_1 = \frac{1}{4} \times 9.5 + \frac{3}{4} \times 9.6 = 9.575\,\text{kg}.$$

Similarly, the upper quartile is the $3(50+1)/4 = 38\frac{1}{4}$th smallest observation, that is, a quarter of the way between the 38th and 39th smallest observations. As both of these observations are 10.6 kg, we have

$$Q_3 = 10.6\,\text{kg}.$$

Therefore, the interquartile range is

$$\text{IQR } = Q_3 - Q_1 = 10.6 - 9.575 = 1.025\,\text{kg}.$$

8. The sample standard deviation is best calculated either using MINITAB or your calculator in SD mode. However we can also do the calculation the old way. We find

$$\sum_{i=1}^{50} x_i^2 = 5157.54.$$

Therefore the sample variance is

$$\begin{aligned} s^2 &= \frac{1}{n-1}\left\{\sum_{i=1}^{n} x_i^2 - n\bar{x}^2\right\} \\ &= \frac{5157.54 - 50 \times 10.116^2}{49} \\ &= 0.834024 \end{aligned}$$

The sample standard deviation is

$$s = \sqrt{s^2} = \sqrt{0.834024} = 0.913\,kg.$$

Note that this is the value obtained on calculators using the s or σ_{n-1} button and not the σ or σ_n button.

9. The box and whisker plot should look like this: