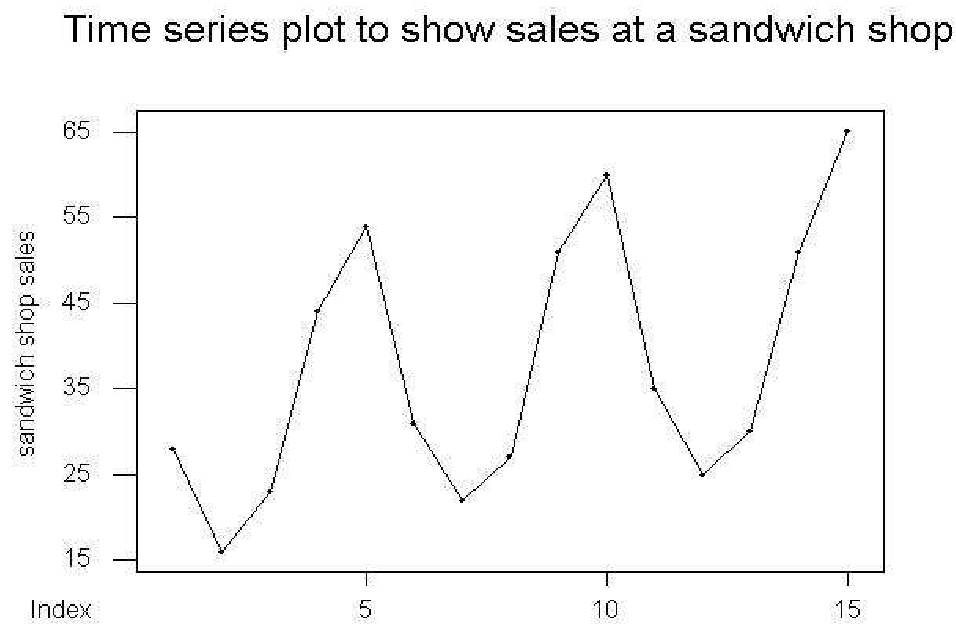
Chapter 8: Model solutions

(a) The time series plot below was drawn in `Minitab` – if you've done yours by hand, it should look similar!



(b) The time series plot shows there to be a trend in sales at the sandwich shop, with sales increasing through time (positive/direct trend). The plot also shows clear seasonal variations, with sales being highest at the end of the week and lowest on Tuesdays.

(c) The five–observation cycle moving averages for sales at the sandwich shop are given in the table below – note that there's no moving averages associated with the first two, and last two, observations.

	Mon	Tues	Wed	Thurs	Fri
Week beginning 21/02/05	*	*	33	33.6	34.8
Week beginning 28/02/05	35.6	37	38.2	39	39.6
Week beginning 7/03/05	40.2	40.2	41.2	*	*

(d) We can estimate the simple linear regression model

$$Y = \alpha + \beta T + \epsilon$$

with

$$\hat{\beta} = \frac{S_{TY}}{S_{TT}} \quad \text{and}$$
$$\hat{\alpha} = \bar{y} - \hat{\beta}\bar{t},$$

where

$$
\begin{aligned}
S_{TY} &= \left(\sum ty\right) - n\bar{t}\bar{y} \qquad \text{and} \\
S_{TT} &= \left(\sum t^2\right) - n\bar{t}^2.
\end{aligned}
$$

The easiest way to calculate these is to draw up a table!

t	y (moving averages)	ty	t^2
3	33	99	9
4	33.6	134.4	16
5	34.8	174	25
6	35.6	213.6	36
7	37	259	49
8	38.2	305.6	64
9	39	351	81
10	39.6	396	100
11	40.2	442.2	121
12	40.2	482.4	144
13	41.2	535.6	169
88	412.4	3392.8	814

Thus,

$$
\begin{aligned}
\bar{t} &= \frac{88}{11} \\
&= 8, \qquad \text{and} \\
\\
\bar{y} &= \frac{412.4}{11} \\
&= 37.49.
\end{aligned}
$$

So we have

$$
\begin{aligned}
S_{TY} &= \left(\sum ty\right) - n\bar{t}\bar{y} \\
&= 3392.8 - 11 \times 8 \times 37.49 \\
&= 93.68 \qquad \text{and} \\
\\
S_{TT} &= \left(\sum t^2\right) - n \times \bar{t}^2 \\
&= 814 - 11 \times 8 \times 8 \\
&= 110.
\end{aligned}
$$

Therefore

$$\begin{aligned}\hat{\beta} &= \frac{S_{TY}}{S_{TT}} \\ &= \frac{93.68}{110} \\ &= 0.852, \qquad \text{and}\end{aligned}$$

$$\begin{aligned}\hat{\alpha} &= \bar{y} - \hat{\beta}\bar{t} \\ &= 37.49 - 0.852 \times 8 \\ &= 30.674.\end{aligned}$$

Thus, the regression equation for our linear trend is

$$Y = 30.674 + 0.852T + \epsilon.$$

(e) The seasonal deviations are found by subtracting the moving averages from the original observations; these are given in the table below, along with the seasonal means.

	Mon	Tues	Wed	Thurs	Fri
Week beginning 21/02/05	*	*	–10	10.4	19.2
Week beginning 28/02/05	–4.6	–15	–11.2	12	20.4
Week beginning 7/03/05	–5.2	–15.2	–11.2	*	*
Seasonal means	-4.9	–15.1	–10.8	11.2	19.8

(f) The overall mean is given by

$$\frac{-10 + 10.4 + 19.2 + \ldots - 11.2}{11} = -0.94545,$$

and so the seasonal effects are

$$\begin{aligned}
\hat{s}_1 &= -4.9 - (-0.94545) \\
&= -3.55455 \\[1em]
\hat{s}_2 &= -15.1 - (-0.94545) \\
&= -14.15455 \\[1em]
\hat{s}_3 &= -10.8 - (-0.94545) \\
&= -9.85455 \\[1em]
\hat{s}_4 &= 11.2 - (-0.94545) \\
&= +12.14545 \qquad \text{and} \\[1em]
\hat{s}_5 &= 19.8 - (-0.94545) \\
&= +20.74545.
\end{aligned}$$

Notice that $\hat{s}_1 + \hat{s}_2 + \hat{s}_3 + \hat{s}_4 + \hat{s}_5 = -3.55455 - 14.15455 - 9.85455 + 12.14545 + 20.74545 = 5.32725$, and so since these do not sum to zero we have to adjust each of the seasonal effects. We do this by finding the mean of the seasonal effects, and then subtracting the mean from each of the effects, i.e.

$$\begin{aligned}
\text{mean of seasonal effects} &= \frac{-3.55455 - 14.15455 - 9.85455 + 12.14545 + 20.74545}{5} \\
&= \frac{5.32725}{5} \\
&= 1.06545.
\end{aligned}$$

Thus, the adjusted seasonal effects are now

$$\begin{aligned}\hat{s}_1 &= -3.55455 - 1.06545\\ &= -4.62\end{aligned}$$

$$\begin{aligned}\hat{s}_2 &= -14.15455 - 1.06545\\ &= -15.22\end{aligned}$$

$$\begin{aligned}\hat{s}_3 &= -9.85455 - 1.06545\\ &= -10.92\end{aligned}$$

$$\begin{aligned}\hat{s}_4 &= 12.14545 - 1.06545\\ &= 11.08 \qquad \text{and}\end{aligned}$$

$$\begin{aligned}\hat{s}_5 &= 20.74545 - 1.06545\\ &= 19.68.\end{aligned}$$

Now the seasonal effects sum to zero, as required!

(g) We can forecast sales for Monday 14th March using the regression equation in part (d) and the seasonal effect for Monday ($\hat{s}_1$) obtained in part (f). Thus

$$\begin{aligned}Y &= 30.674 + 0.852 \times 16\\ &= 44.306,\end{aligned}$$

and so the "full" forecast, found by adding on the seasonal effect for Monday, is

$$44.306 - 4.62 \quad = \quad £39.686,$$

i.e. £$39,686$.

Similarly, for Tuesday 15th March we have

$$\begin{aligned}Y &= 30.674 + 0.852 \times 17\\ &= 45.158,\end{aligned}$$

and the "full" forecast is

$$45.158 - 15.22 \quad = \quad £29.939,$$

i.e. £$29,939$.