Chapter 3: Model solutions

1. Very similar to the example on page 26 of the lecture notes!

2. (a) this is a one–sample test, as we are comparing the mean flight time of EasyAir's flights between Glasgow and Cairo with an advertised value (5 hours = 300 minutes). Thus, the null hypothesis is

$$H_0 \quad : \quad \mu = 300 \text{ mins.}$$

Since the rival company, RyanJet, suspect EasyAir of false advertising, we might want to test the one–tailed alternative

$$H_1 \quad : \quad \mu > 300 \text{ mins.}$$

Since the population variance is unknown, the test statistic is

$$\begin{aligned} t &= \frac{|\bar{x} - \mu|}{\sqrt{s^2/n}} \\ &= \frac{|310 - 300|}{\sqrt{20^2/20}} \\ &= \frac{10}{\sqrt{400/20}} \\ &= \frac{10}{4.472} \\ &= 2.236. \end{aligned}$$

We use t–tables to obtain our p–value for this hypothesis test. Now $\nu = n - 1 = 20 - 1 = 19$, and using this, from table 2.3, we get:

Significance level	10%	5%	1%
Critical value	1.328	1.729	2.539

Our test statistic $t = 2.236$ lies between the two critical values of 1.729 and 2.539, and so our p–value lies between 1% and 5%.

We conclude that there is moderate evidence against the bull hypothesis, and so we reject H_0 and accept the alternative H_1; it appears that EasyAir's flights are, on average, longer than the advertised time of 5 hours (or 300 minutes) between Glasgow and Cairo. So RyanJet's suspicions *are* supported!

(b) We now want to compare the average flight time of RyanJet's flights with EasyAir's on this route. To summarise, we have

EasyAir	RyanJet
$n_1 = 20$	$n_2 = 23$
$\bar{x}_1 = 310$	$\bar{x}_2 = 304$
$s_1 = 20$	$s_2 = 22$

Remember – for a two–sample test, our null hypothesis is

$$H_0 \quad : \quad \mu_1 = \mu_2,$$

i.e. there's no difference between the population mean flight times of the two airlines. Since the question says "Are RyanJet's flights shorter than EasyAir's?", we need to test the one–tailed alternative hypothesis

$$H_1 \quad : \quad \mu_1 > \mu_2$$

(i.e. EasyAir's are longer).

Since the population variances are unknown, we use the test statistic

$$\begin{aligned} t &= \frac{|\bar{x}_1 - \bar{x}_2|}{s \times \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}, \qquad \text{i.e.} \\ t &= \frac{|310 - 304|}{s \times \sqrt{\frac{1}{20} + \frac{1}{23}}} \\ &= \frac{6}{s \times 0.306}. \end{aligned}$$

Now remember that s is the "pooled standard deviation", and is equal to

$$s = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}.$$

Note that the calculation of s requires s_1^2 and s_2^2, i.e. the sample *variances*. What we have are the sample *standard deviations*. Thus

$$\begin{aligned} s_1^2 &= 20 \times 20 \\ &= 400, \end{aligned}$$

and

$$\begin{aligned} s_2^2 &= 22 \times 22 \\ &= 484. \end{aligned}$$

Thus,

$$\begin{aligned} s &= \sqrt{\frac{19 \times 400 + 22 \times 484}{20 + 23 - 2}} \\ &= \sqrt{\frac{18248}{41}} \\ &= \sqrt{445.073} \\ &= 21.097. \end{aligned}$$

So now we can calculate t!

$$\begin{aligned} t &= \frac{6}{21.097 \times 0.306} \\ &= \frac{6}{6.456} \\ &= 0.929. \end{aligned}$$

Since the population variances are unknown, we need to use t tables (table 2.3 in the lecture notes) to obtain our p–value. The degrees of freedom, $\nu = n_1 + n_2 - 2 = 20 + 23 - 2 = 41$. Looking at table 2.3, we see that the degrees of freedom only goes up to 29; thus, since we have a large sample size, we use the "infinity" (or ∞) row of the table. This gives the following values:

Significance level	10%	5%	1%
Critical value	1.282	1.645	2.326

Since our test statistic $t = 0.929$ lies to the left of the above table, our p–value is bigger than 10%.

We can conclude that there is no evidence against the null hypothesis, and so we should *retain* H_0. It appears that, on average, there is no significant difference between the flight times of EasyAir and RyanJet between Glasgow and Cairo.

3. This is one–sample test, as we are comparing the mean of a single sample with a hypothesised value. Our null hypothesis is

$$H_0 \quad : \quad \mu = 19.5.$$

Since the question wants us to find out if younger students are favoured, we test the null hypothesis against the one–tailed alternative

$$H_1 \quad : \quad \mu < 19.5.$$

Since the population variance is unknown, we use the test statistic is

$$t \quad = \quad \frac{|\overline{x} - \mu|}{\sqrt{s^2/n}}.$$

Now in this example, $\bar{x}$ and s^2 aren't given, so we have to find the mean and variance by hand (or use a calculator!). Thus,

$$\begin{aligned} \overline{x} &= \frac{17.9 + 18.2 + \ldots + 17.8}{7} \\ &= 18.36; \end{aligned}$$

similarly,

$$\begin{aligned} s^2 &= \frac{(17.9-18.36)^2+(18.2-18.36)^2+\ldots+(17.8-18.36)^2}{6} \\ &= 1.02. \end{aligned}$$

So the test statistic is

$$\begin{aligned} t &= \frac{|18.36-19.5|}{\sqrt{1.02/7}} \\ &= \frac{1.14}{0.382} \\ &= 2.98. \end{aligned}$$

Since the population variance is unknown, we must use tables of values for the t–distribution to obtain our p–value. The degrees of freedom, $\nu = n-1 = 7-1 = 6$; thus, the critical values (from table 2.3 in the notes, for a one–tailed test) are:

Significance level	10%	5%	1%
Critical value	1.440	1.943	3.143

Our test statistic, $t = 2.98$, lies in between the critical values 1.943 and 3.143, and so our p–value lies between 1% and 5%.

We conclude that there is moderate evidence against the null hypothesis, and so we reject H_0 in favour of H_1. It appears that, on average, younger students *are* being favoured.

4. This is a two–sample test. Our null hypothesis is

$$H_0 \quad : \quad \mu_1 = \mu_2,$$

where the subscripts 1 and 2 denote streakers and non–streakers (respectively). Since the question states "are streakers more extrovert than non–streakers?", we test against the one–tailed alternative

$$H_1 \quad : \quad \mu_1 > \mu_2.$$

Since neither population variance is known, we use the test statistic

$$\begin{aligned} t &= \frac{|\bar{x}_1-\bar{x}_2|}{s\times\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}, \qquad \text{i.e.} \\ t &= \frac{|15.26-13.90|}{s\times\sqrt{\frac{1}{19}+\frac{1}{19}}} \\ &= \frac{1.36}{s\times 0.324}. \end{aligned}$$

Now remember that s is the "pooled standard deviation", and is equal to

$$\begin{aligned} s &= \sqrt{\frac{(n_1-1)s_1^2+(n_2-1)s_2^2}{n_1+n_2-2}} \\ &= \sqrt{\frac{18\times 2.62+18\times 4.11}{19+19-2}} \\ &= \sqrt{\frac{47.16+73.98}{36}} \\ &= 1.834. \end{aligned}$$

Hence

$$\begin{aligned} t &= \frac{1.36}{1.834\times 0.324} \\ &= 2.289. \end{aligned}$$

Since neither population variances are known, we use t–tables to obtain our p–value. the degrees of freedom $\nu = 19+19-2 = 36$. As before, you should notice that table 2.3 only goes down to $\nu = 29$, and so we use the ∞ row. Doing so gives the following values:

Significance level	10%	5%	1%
Critical value	1.282	1.645	2.326

Our test statistic $t = 2.289$ lies in between the critical values 1.645 and 2.326, and so our p–value lies between 1% and 5%.

We conclude that there is moderate evidence against the null hypothesis, and so we should reject H_0 in favour of H_1; it appears that, on average, streakers *are* more extrovert than non–streakers.

5. This is a two–sample test – we want to compare the average coverage of two types of paint. Our null hypothesis is

$$H_0 \ : \ \mu_1 = \mu_2,$$

that is, there is no difference in the mean coverage of each type of paint. Since the question doesn't ask us to determine which brand of paint has the most or least coverage, we use a general, two–sided alternative, i.e.

$$H_1 \ : \ \mu_1 \neq \mu_2.$$

In this question, the population standard deviations are both *known*; For "Wilko's Best", we have $\sigma_1 = 31$ and for "Dulor" we have $\sigma_2 = 26$. Thus, the population *variances* are $\sigma_1^2 = 31\times 31 = 961$ and $\sigma_2^2 = 26\times 26 = 676$.

The test statistic is

$$z = \frac{|\bar{x}_1 - \bar{x}_2|}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}.$$

Now we don't know the sample means; however, we can calculate these from the data. For "Wilko's Best", we have

$$\bar{x}_1 = 545.5;$$

similarly, for "Dulor", we have

$$\bar{x}_2 = 521.$$

Thus,

$$\begin{aligned} z &= \frac{|545.5 - 521|}{\sqrt{\frac{961}{8} + \frac{676}{10}}} \\ &= \frac{24.5}{13.701} \\ &= 1.788. \end{aligned}$$

We use standard normal tables (table 2.2 in the notes) to obtain our p–value. For a two–tailed test, this gives:

Significance level	10%	5%	1%
Critical value	1.645	1.96	2.576

Since our test statistic $z = 1.788$ lies in between the critical values 1.645 and 1.96, our p–value lies between 5% and 10%.

In conclusion, we can say that there is only *slight* evidence against the null hypothesis, and so we should *retain* H_0. It appears that, on average, both brands of paint give the same coverage.