Chapter 2 exercises: Model solutions

1. First of all, we set up our null and alternative hypotheses, which are

$$\begin{aligned} H_0 &: \mu = 250 \qquad \text{versus} \\ H_1 &: \mu \neq 250. \end{aligned}$$

We now calculate the test statistic. Since the process variance is *known*, the test statistic is found using

$$\begin{aligned} z &= \frac{|\overline{x} - \mu|}{\sqrt{\sigma^2/n}} \qquad \text{i.e.} \\ z &= \frac{|240 - 250|}{\sqrt{400/100}} \\ &= \frac{10}{\sqrt{4}} \\ &= \frac{10}{2} \\ &= 5. \end{aligned}$$

To obtain our p–value, we use standard normal distribution tables (since the population variance is know in this case). Thus, referring to table 2.2 gives

Significance level	10%	5%	1%
Critical value	1.645	1.96	2.576

Since our test statistic of $z = 5$ lies to the right–hand–side of the last critical value of 2.576, then our p–value is smaller than 1%.

In conclusion, we have strong evidence against the null hypothesis (using table 2.1 to interpret our p–value). Thus we reject H_0 in favour of the laternative H_1; it appears that the population mean is *not* equal to 250ml, i.e. the filling machine is *not* consistent with the stated weight of 250ml.

2. As before, we set up our null and alternative hypotheses, and state our significance level. Our hypotheses are

$$\begin{aligned} H_0 &: \mu = 100 \qquad \text{versus} \\ H_1 &: \mu < 100. \end{aligned}$$

We use a one–tailed test because the question asks "are the bolts being made too short?", and there is evidence from our sample that the average length of bolts *is* less than 100mm.

Since the process variance is *unknown*, the test statistic is found as

$$\begin{aligned} t &= \frac{|\overline{x} - \mu|}{\sqrt{s^2/n}} \qquad \text{i.e.} \\ t &= \frac{|97.5 - 100|}{\sqrt{150/50}} \\ &= \frac{2.5}{\sqrt{3}} \\ &= \frac{2.5}{1.732051} \\ &= 1.443. \end{aligned}$$

We now obtain our p–value. Since the population variance is *unknown*, we must refer to the table of values for the t–distribution (i.e. table 2.3) on $\nu = n - 1 = 50 - 1 = 49$ degrees of freedom. However, this corresponds to a "large" sample, and so we use the ∞ row in table 2.3. Thus, for a one–tailed test, we get:

Significance level	10%	5%	1%
Critical value	1.282	1.645	2.326

Our test statistic of $t = 1.443$ lies in between the critical values 1.282 and 1.645, and so our p–value lies between 5% and 10%.

Thus, in conclusion, we have only slight evidence against the null hypothesis and so we should *retain* H_0. There is insufficient evidence to suggest that the population mean differs from 100mm – i.e. the bolts are not being made too short!

3. Again, we set up our null and alternative hypotheses, and state our significance level. Our hypotheses are

$$\begin{aligned} H_0 &: \mu = 37.5 \qquad \text{versus} \\ H_1 &: \mu > 37.5. \end{aligned}$$

We use a one–tailed test because the question asks "are the staff working *more* than a standard week?".

Since the process variance is *unknown*, the test statistic is found using

$$t = \frac{|\bar{x} - \mu|}{\sqrt{s^2/n}}.$$

However, this time, $\bar{x}$ and s^2 are not given directly, and we must calculate these quantities from the raw data. Thus,

$$\begin{aligned}\bar{x} &= \frac{35 + 40 + 45 + \ldots + 32}{10} \\ &= \frac{385}{10} \\ &= 38.5.\end{aligned}$$

Similarly,

$$\begin{aligned}s^2 &= \frac{1}{n-1}\sum_{i=1}^{10}(x_i - \bar{x})^2 \\ &= 14.055.\end{aligned}$$

[Remember, it might save time to use the `Stats` *mode to compute these quantities on a calculator]*

Thus,

$$\begin{aligned}t &= \frac{|38.5 - 37.5|}{\sqrt{14.055/10}} \\ &= \frac{1}{\sqrt{1.4055}} \\ &= \frac{1}{1.185538} \\ &= 0.843.\end{aligned}$$

To obtain the p–value for our test, we need to refer to values from the t–distribution, since in this example the population variance is unknown. Thus, referring to table 2.3 with $\nu = n - 1 = 10 - 1 = 9$ and for a one–tailed test, we get

Significance level	10%	5%	1%
Critical value	1.383	1.833	2.821

Since our test statistic $t = 0.843$ lies to the left of the first critical value in the above table, our p–value is bigger than 10%.

Thus, in conclusion, we can say that there is no evidence against the null hypothesis, and so we *retain* H_0. There is insufficient evidence to suggest that the population mean differs from 37.5 hours; i.e. the staff are not working more than a standard week!