Weeks 1–4 (Data collection and summaries)

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- How to *collect* data

Where we are in the course

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- How to *summarise* data

Where we are in the course

Weeks 1–4 (Data collection and summaries)

- How to *collect* data
- How to *summarise* data
  - Tabular
  - Graphical
  - Numerical (location and spread)

Weeks 5–7 (Probability)

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- Introduction to probability
  - Interpretations of probability
  - Laws of probability

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Don't forget:

- Computer practical this week in place of tutorial!
- CBA2 deadline midnight this Friday!

Weeks 8–11 (Probability models)

Where we are in the course

Weeks 8–11 (Probability models)

- Models for discrete data
  - The Binomial distribution
  - The Poisson distribution

Where we are in the course

Weeks 8–11 (Probability models)

- Models for discrete data
  - The Binomial distribution
  - The Poisson distribution
- Models for continuous data
  - The Normal distribution
  - The Uniform distribution
  - The exponential distribution

Die/Dice/Coins/socks and more crap examples

- Sorry about the rubbish examples I've used for probability!
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- Sorry about the rubbish examples I've used for probability!
- **But** they do keep things simple and test your understanding of the basics
- More interesting (and relevant) stuff is just around the corner...
- ...but that often means more difficult maths!

Die/Dice/Coins/socks and more crap examples

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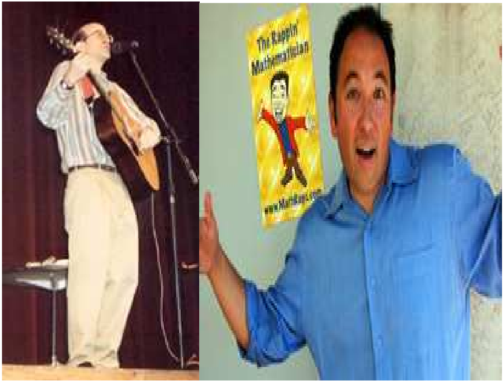

The **Mathemusician**

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The **Mathemusician** The **Rappin' Mathematician** play

What are the chances,
The likely outcomes?
What probably will happen?
That's probability.

When you flip a coin,
There are just two chances,
Heads or tails, two possibilities.
When you flip a coin, just two chances
Heads or tails, that's probability.

When you roll a die
There are just six chances,
1 2 3 4 5 or 6.
When you roll a die, just 6 chances
One-sixth probability.

play

Chorus:
Break it down, break it down, check the thesis,
a fraction is part of something broken into pieces.

Any measurement that's not a whole unit is called a fraction.
This is how we're doin' it.

Rap Verse One:
I've got a little story and its all about fractions,
So pay attention and follow the action!

Get the idea....Now what we gonna do?! ...

Lecture 7

DECISION MAKING USING PROBABILITY

Decision–making using probability

In this lecture, we look at how we can use probability in order to aid management **decision–making**.

Expected Monetary Value

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which is quite good, and so surely we should launch the product? It looks promising!

Expected Monetary Value

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Such financial considerations could outweigh the high probability of success alone.

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The **Expected Monetary Value** (EMV) of a single event is simply the probability of that event multiplied by the monetary value of that outcome.

Example 1

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In other words, if you repeated this bet a large number of times, overall you would come out, on average, 38 pence better off per bet.

Example 2

Consider another bet. When rolling a die,

- if it's a six you have to *pay* £5
- if it's any other number you *receive* £2.50

Would you take on this bet?

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Probability	Financial outcome
$P(6) = 1/6$	$-£5$
$P(\text{Not a 6}) = 5/6$	£2.50

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Therefore, in the long run, this would be a bet to take on as it has a **positive expected monetary value**.

Example 3: The National Lottery

In a recent lottery draw, the prizes were

Number of balls matched	Probability	Prize
6	0.000000071	£2.4M
5 plus bonus	0.000000429	£240K
5	0.000018449	£3K
4	0.000968619	£100
3	0.0177	£10
< 3	0	£0

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Therefore, a fair price for a ticket in this particular lottery is around 62p.

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Therefore, a fair price for a ticket in this particular lottery is around 62p. The difference between this and the standard £1 charge for a ticket goes to "good causes" and, of course, **Camelot's profits**.

In general, the expected monetary value of a project or bet is given by the formula

$$EMV = \sum P(\text{Event}) \times \text{ Monetary value of Event}$$

where the sum is over all possible events.

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The *EMV* of a project can be used as a **decision criterion** for choosing between different projects and has applications in a large number of situations.

Example 4

A small company is trying to decide how to launch a new and innovative product.

It could go for a direct approach, launching onto the whole of the domestic market through traditional distribution channels, or it could launch only on the internet.

A third option exists where the product is licensed to a larger company through the payment of a licence fee irrespective of the success of the product.

How should the company launch the product?

Example 4

The company has done some initial market research and the managing director believes the probability of the product being successful can be classed into three categories:

High **Medium** **Low**

She thinks that these categories will occur with probabilities 0.2, 0.35 and 0.45 respectively and her thoughts on the likely profits (in £K) to be earned in each plan are

	High	**Medium**	**Low**
Direct	100	55	-25
Internet	46	25	15
Licence	20	20	20

The *EMV* of each plan can be calculated as follows:

$$EMV(\textbf{Direct}) = 0.2 \times 100 + 0.35 \times 55 + 0.45 \times (-25) = \text{£}28\text{K}$$
$$EMV(\textbf{Internet}) = 0.2 \times 46 + 0.35 \times 25 + 0.45 \times 15 = \text{£}24.7\text{K}$$
$$EMV(\textbf{Licence}) = 0.2 \times 20 + 0.35 \times 20 + 0.45 \times 20 = \text{£}20\text{K}.$$

On the basis of expected monetary value, the best choice is the Direct approach.

Decision trees

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In a decision tree the first node is always a decision node. There may also be other decision nodes. If there is another decision node then we evaluate the options there and choose the best and the expected value of this option becomes the expected value of the branch leading to the decision node.

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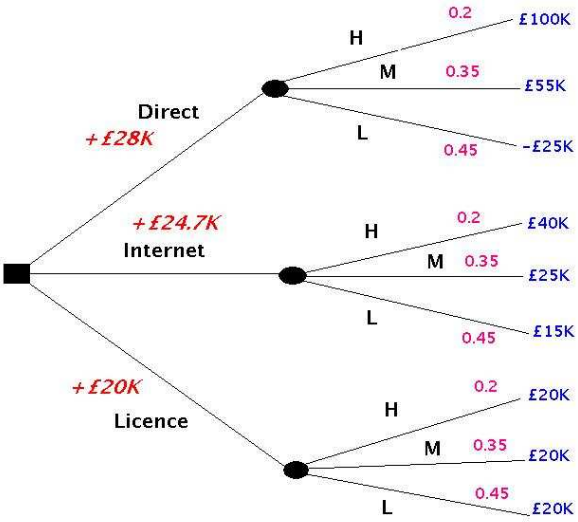

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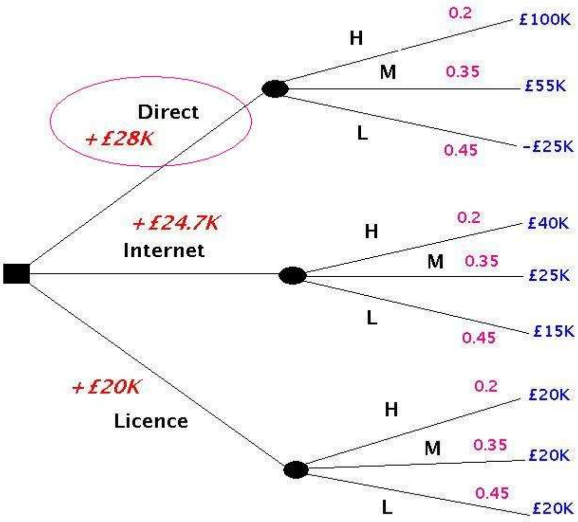


A more complicated example

The manager of a small I.T. sales company has the opportunity to buy a fixed quantity of a new type of soundcard for home PCs which they can then offer for sale to clients.

The decision to buy the product and offer it for sale would involve a fixed cost of £200,000. The number of soundcards that would be sold is uncertain, but the manager's prior beliefs are expressed as follows.

- Sales will be "poor" with probability 0.2; this will result in an income of £100,000.
- Sales will be "moderate" with probability 0.5; this will result in an income of £220,000.
- Sales will be "good" with probability 0.3; this will result in an income of £350,000.

A more complicated example

For an additional fixed cost of £30,000, market research can be conducted to aid the decision–making process.

The outcome of the market research can be either positive or negative, with probabilities 0.58 and 0.42 respectively. Knowing the outcome of the market research changes the probabilities for the main sales project as follows:

	Main sales probabilities		
Market research	Poor	Moderate	Good
Positive	0.15	0.45	0.4
Negative	0.6	0.35	0.05

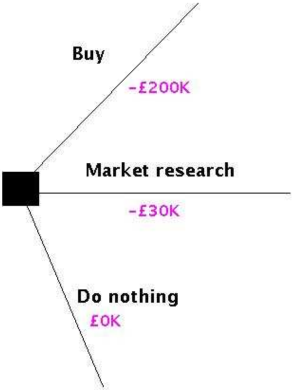
A more complicated example

The manager will make decisions based on expected monetary value.

(a) Draw a decision tree for this problem.

(b) Use expected monetary value to determine the optimal course of action for the company.

A more complicated example



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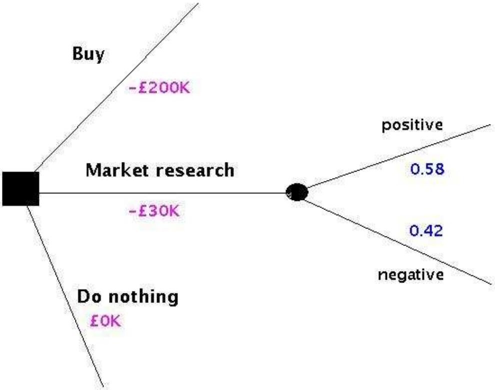

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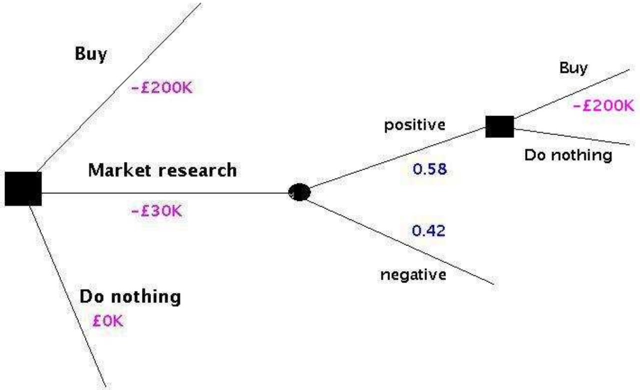

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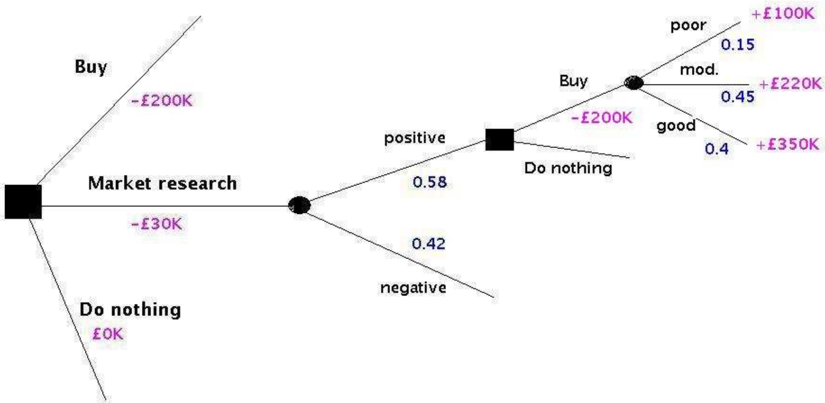

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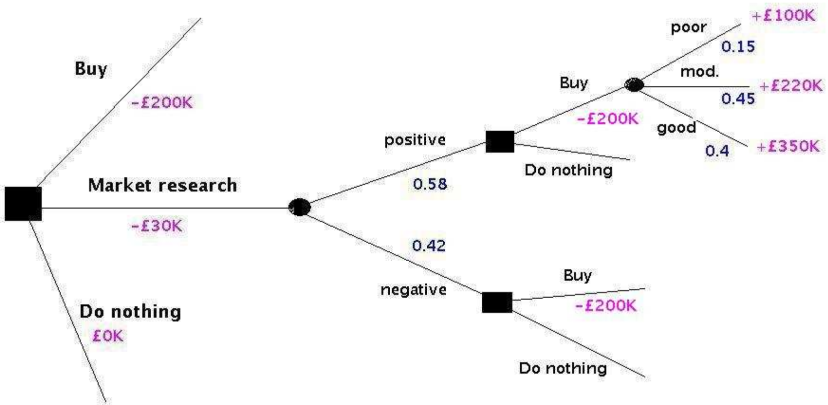

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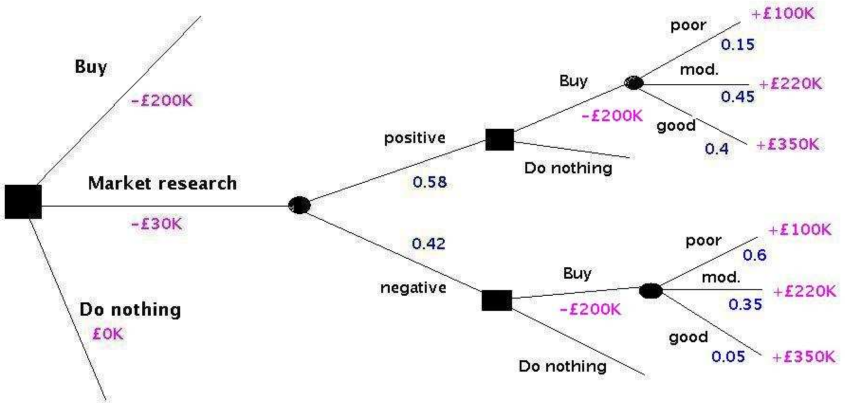

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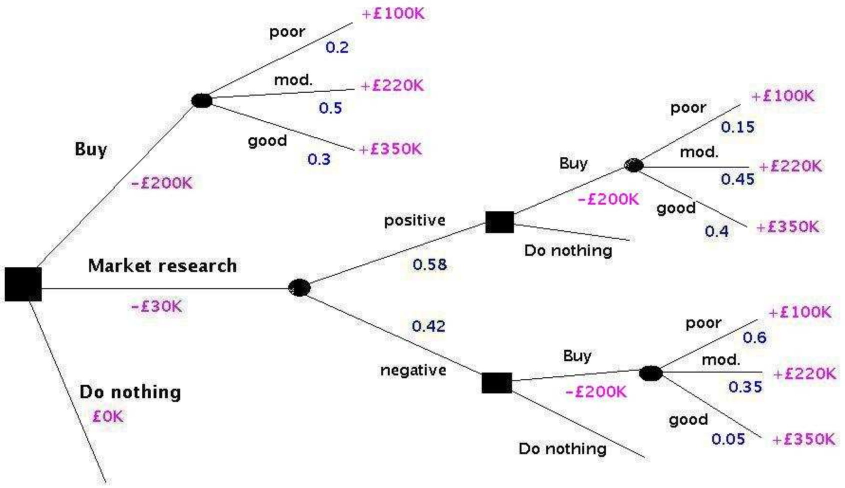

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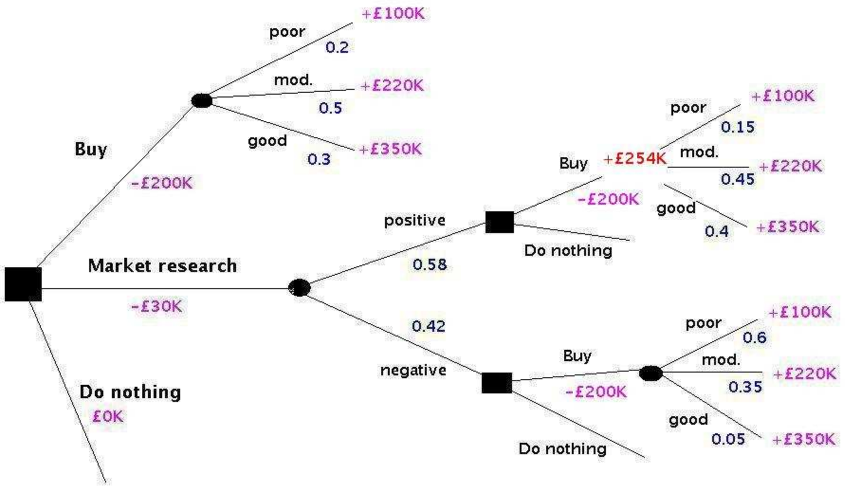

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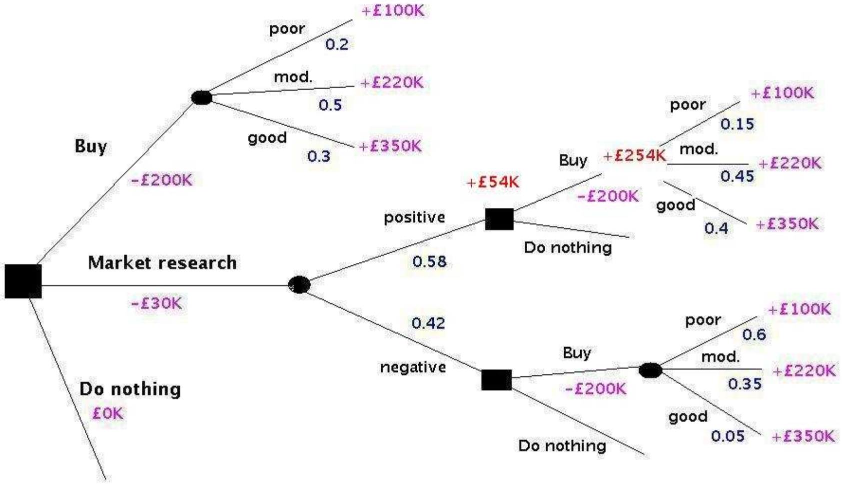

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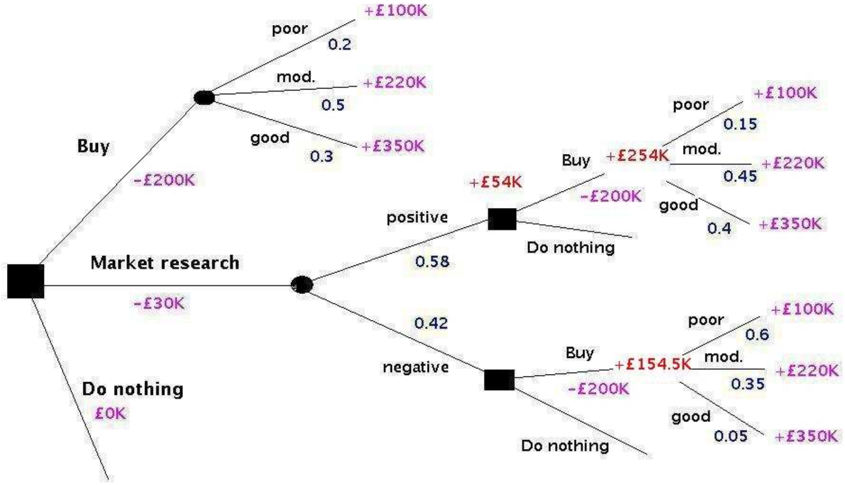

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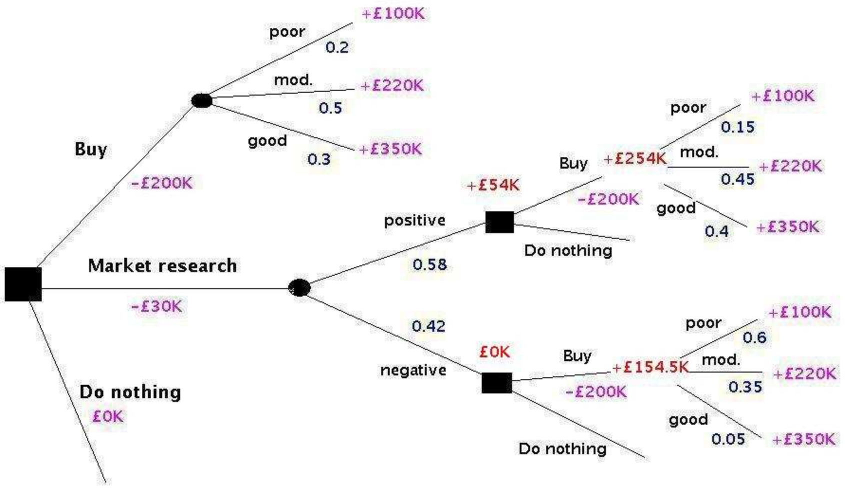

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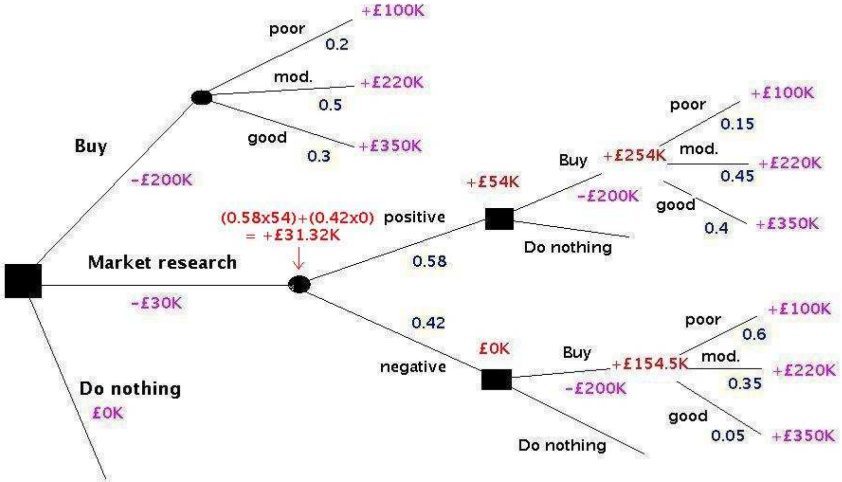

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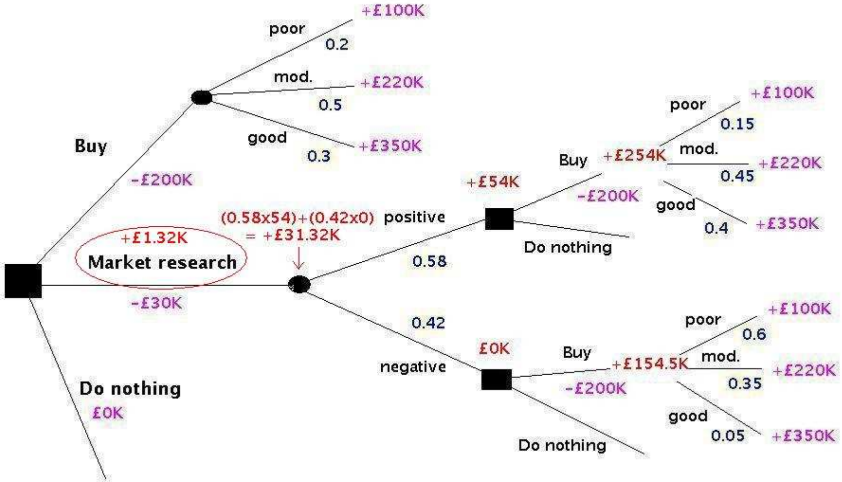

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