Lecture 3

HYPOTHESIS TESTS FOR TWO MEANS

Announcements

CBA4 goes live in practice mode this week – exam mode next week

Assignment 1 feedback

Mean	St. dev.	Median	IQR	95% CI	Missing
89.9	10.2	94.0	11.1	(89.1, 90.9)	27

Introduction

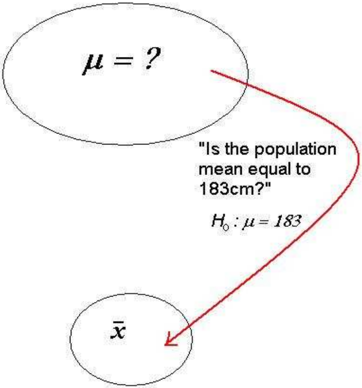
Last week you were introduced to the concept of hypothesis testing in statistics, and we considered hypothesis tests for the mean if we have a **single** sample drawn from a **single** population.

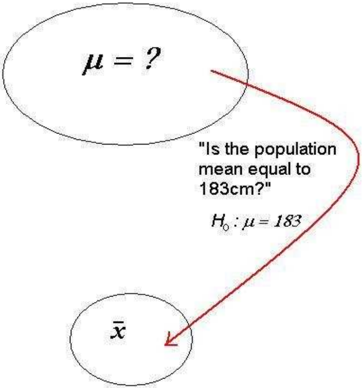
If we have **two** independent random samples from **two** populations, we can compare the two sample means in a test for two means (*c.f.* comparing one sample mean to a *proposed value* in the one–sample case).

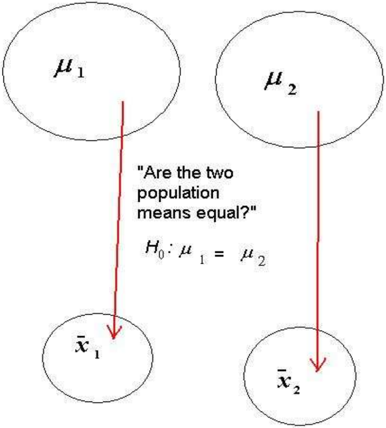
$$\mu = ?$$

"Is the population mean equal to 183cm?"

$$H_0 : \mu = 183$$


$\mu = ?$
"Is the population mean equal to 183cm?"
$H_0 : \mu = 183$
$\bar{x}$


$\mu = ?$
"Is the population mean equal to 183cm?"
$H_0 : \mu = 183$
$\bar{x}$


μ_1
μ_2
"Are the two population means equal?"
$H_0 : \mu_1 = \mu_2$
$\bar{x}_1$
$\bar{x}_2$

We use the same framework for hypothesis testing as for the one–sample tests:

1. State the **null hypothesis**, H_0;
2. State the **alternative hypothesis**, H_1;
3. Calculate a **test statistic**;
4. Find the **p–value**, and
5. Use table 2.1 to form your **conclusions**.

However, the calculations required for the test statistic in step 3 are slightly different.

Testing two means

Recall that, in the test for one mean, there were two cases: population variance (σ^2) known and population variance unknown.

Similarly, when comparing two means, we can consider

- **case 1:** both population variances **known** and
- **case 2:** both population variances **unknown**.

Both population variances (σ_1^2 and σ_2^2) known

1. **State the null hypothesis**

   This time, the null hypothesis is

   $$H_0 \quad : \quad \mu_1 = \mu_2,$$

   i.e. the two population means are equal.

2. **State the alternative hypothesis**

   We usually test against the (two–tailed) alternative:

   $$H_1 \quad : \quad \mu_1 \neq \mu_2,$$

   i.e. the population means *are not* equal. However, we might use the one–tailed alternatives:

   $$\begin{aligned} H_1 \quad &: \quad \mu_1 > \mu_2, \qquad \text{or} \\ H_1 \quad &: \quad \mu_1 < \mu_2. \end{aligned}$$

3. **Calculate the test statistic**

   The test statistic for a two–sample test (when both population variances are known) is

$$z = \frac{|\bar{x}_1 - \bar{x}_2|}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

4. **Find the p–value**

   This is found from statistical tables; since, in this case, both population variances σ_1^2 and σ_2^2 are *known*, we refer to standard normal tables.

   As before, we find a range for our p–value by comparing our test statistic to the 10%, 5% and 1% critical values.

5. **Form a conclusion**
   Exactly the same again! Use table 2.1 to help you decide what to do! Word your conclusions in the context of the original question.

Example (page 25)

Before a training session for call centre employees a sample of 50 calls to the call centre had an average duration of 5 minutes, whereas after the training session a sample of 45 calls had an average duration of 4.5 minutes.

The population variance is known to have been 1.5 minutes before the course and 2 minutes afterwards.

Has the course been effective?

Example (page 25)

Steps 1 and 2 *(hypotheses)*
We test

$$\begin{aligned} H_0 &: \mu_B = \mu_A \qquad \text{against} \\ H_1 &: \mu_B \neq \mu_A \end{aligned}$$

Example (page 25)

Step 3 *(test statistic)*

Since both population variances are known, we use

$$
\begin{aligned}
z &= \frac{|\bar{x}_B - \bar{x}_A|}{\sqrt{\frac{\sigma_B^2}{n_B} + \frac{\sigma_A^2}{n_A}}} \\
&= \frac{|5 - 4.5|}{\sqrt{\frac{1.5}{50} + \frac{2}{45}}} \\
&= 1.833
\end{aligned}
$$

Example (page 25)

Step 4 *(p–value)*

We used a **two–tailed** alternative; using table 2.2, we get:

Significance level	**10%**	**5%**	**1%**
Critical value	1.645	1.96	2.576

Since $z = 1.833$ lies between 1.645 and 1.96, our p–value lies between 5% and 10%.

Example (page 25)

Step 5 *(conclusion)*
Using table 2.1 (last week), this means that:

- we have **slight** evidence against H_0
- This is not small enough to reject H_0, and so we **retain** H_0
- There is insufficient evidence to suggest that the training has had any affect on the average duration of a call.

Both population variances (σ_1^2 and σ_2^2) unknown

In the more likely situation where the population variances are unknown, the test statistic becomes

$$t = \frac{|\bar{x}_1 - \bar{x}_2|}{s \times \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}},$$

where s is a "pooled standard deviation", and is found as

$$s = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}.$$

Like before, we have to use t–tables to obtain our critical value; the degrees of freedom is now found as $\nu = n_1 + n_2 - 2$.

Example (page 26)

A company is interested in knowing if two branches have the same level of average transactions. The company sample a small number of transactions and calculates the following statistics:

Shop 1	$\bar{x}_1 = 130$	$s_1^2 = 700$	$n_1 = 12$
Shop 2	$\bar{x}_2 = 120$	$s_2^2 = 800$	$n_2 = 15$

Test whether or not the two branches have (on average) the same level of transactions.

Example (page 26)

Steps 1 and 2 *(hypotheses)*

Our null and alternative hypotheses are:

$$\begin{aligned} H_0 &: \mu_1 = \mu_2 \qquad \text{versus} \\ H_1 &: \mu_1 \neq \mu_2. \end{aligned}$$

Step 3 *(calculating the test statistic)*

Since both population variances are unknown (only the *sample* values are given), the test statistic is

$$t = \frac{|\bar{x}_1 - \bar{x}_2|}{s \times \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}};$$

thus, we first need to obtain the pooled variance s. This is given as

$$\begin{aligned} s &= \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}} \\ &= \sqrt{\frac{11 \times 700 + 14 \times 800}{25}} \\ &= \sqrt{\frac{7700 + 11200}{25}} \\ &= 27.495. \end{aligned}$$

Example (page 26)

Thus,

$$
\begin{aligned}
t &= \frac{|130 - 120|}{27.495 \times \sqrt{\frac{1}{12} + \frac{1}{15}}} \\
&= \frac{10}{27.495 \times \sqrt{0.15}} \\
&= \frac{10}{10.649} \\
&= 0.939.
\end{aligned}
$$

Example (page 26)

Step 4 *(finding the p–value)*

Since both population variances are unknown, we use t–tables to obtain our critical value.

The degrees of freedom is

$$\begin{aligned} \nu &= n_1 + n_2 - 2 \qquad \text{i.e} \\ &= 12 + 15 - 2 \\ & 25. \end{aligned}$$

Under a two–tailed test, and using table 2.3, we get the following critical values:

Significance level	10%	5%	1%
Critical value	1.708	2.060	2.787

Our test statistic $t = 0.939$ lies to the left of the first critical value, and so our p–value is bigger than 10%.

Step 5 *(conclusion)*
Using table 2.1, we see that, since our p–value is larger than 10%, we have no evidence to reject the null hypothesis. Thus, we retain H_0 and conclude that there is no significant difference between the average level of transactions at the two shops.