MAS1403

Quantitative Methods for Business Management

Revision material for Semester 1

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Semester 1 syllabus

Use this checklist to tick things off as you revise them!

- **Collecting and presenting data**
- **Graphical methods for presenting data**
  - stem–and–leaf plots;
  - bar charts and multiple bar charts;
  - histograms.
- **More graphical methods for presenting data**
  - % relative frequency histograms and polygons;
  - ogives (cumulative frequency polygons);
  - pie charts;
  - time series plots;
  - scatterplots.
- **Numerical summaries for data**
  - measures of *location* – mean, median and mode;
  - measures of *spread* – range, IQR and variance/standard deviation;
  - box and whisker plots.
- **Introduction to probability**
- **Conditional probability**
- **Decision–making using probability**
  - Expected Monetary Value
  - Decision trees
- **Discrete probability models**
  - permutations and combinations;
  - probability distributions, expectation and variance;
  - the binomial distribution;
  - the Poisson distribution.
- **Continuous probability models – the Normal distribution**
- **More continuous probability models**
  - the uniform distribution;
  - the exponential distribution;
  - Poisson processes.

1 Collecting and presenting data

1.1 Definitions

The quantities measured in a study are called *random variables* and a particular outcome is called an *observation*. A collection of observations is the *data*. The collection of all possible outcomes is the *population*.

We can rarely observe the whole population. Instead, we observe some sub–set of this called the *sample*. The difficulty is in obtaining a *representative* sample.

All of these definitions can be summarised in a diagram:

The following diagram provides a useful summary of the *type* of data that can be collected:

1.2 Sampling techniques

This section outlines the key concepts, and main pros and cons, of each type of sampling covered in this course.

1. **Simple random sampling**

   – Each element in the population is equally likely to be drawn into the sample.

   – All elements are "put in a hat" and the sample is drawn from the "hat" at random.

   – Advantages – easy to implement; each element has an equal chance of being selected.

   – Disadvantages – often don't have a complete list of the population; not all elements might be equally accessible; it is possible, purely by chance, to pick an unrepresentative sample.

2. **Stratified sampling**

   – We take a simple random sample from each "strata", or group, within the population. The sample sizes are usually proportional to the population sizes.

   – Advantages – sampling within each stratum ensures that that stratum is properly represented in the sample; simple random sampling within each stratum has the advantages of listed under simple random sampling above.

   – Disadvantages – need information on the size and composition of each group; as with s.r.s., we need a list of all elements within each strata.

3. **Systematic sampling**

   – The first element from the population is selected at random, and then every kth item is chosen after this. This type of sampling is often used in a production line setting.

   – Advantages – it's simplicity! – and so it's easy to implement;

   – Disadvantages – not completely random; if there is a pattern in the production process it is easy to obtain a biased sample; only really suited to structured populations.

4. **Multi–stage sampling**

   – The population is divided into geographic areas, and then just one of these areas is selected (probably randomly). A simple random sample, or some other type of sample, is then taken from this area.

   – Advantages – saves time and cost, since sampling is concentrated within one geographic area.

   – Disadvantages – the sample can be biased of the "stages", or "geographic areas", are not carefully defined.

5. **Cluster sampling**

   – Similar to multi–stage sampling, but this time all the elements within the chosen cluster are sampled, and the cluster isn't chosen randomly.

   – Advantages – Fairly easy (and cheap) to implement – we just sample everyone within the chosen cluster!

   – Disadvantages – easy to get a biased sample.

6. **Judgemental sampling**

   – The person interested in obtaining the data decides who should be surveyed; for example, the target population might be 16–25 year old females.

   – Advantages – very focussed and aimed at the target population.

   – Disadvantages – relies on the judgement of the person conducting the questionnaire/survey, and so could include elements not within the target population.

7. **Accessibility sampling**

   – Here, the most easily accessible elements are sampled.

   – Advantages – easy to implement.

   – Disadvantages – prone to bias.

8. **Quota sampling**

   – Similar to stratified sampling, but uses judgemental sampling within each strata instead of random sampling. We sample within each strata until our quotas have been reached.

   – Advantages – results can be very accurate as this technique is very targeted.

   – Disadvantages – the identification of appropriate quotas can be problematic; this sampling technique relies heavily on the judgement of the interviewer.

1.3 Frequency tables

Once we have collected our data, often the first stage of any analysis is to present them in a simple and easily understood way. Tables are perhaps the simplest means of presenting data. You should know how to construct (and read!) simple frequency tables and relative % frequency tables for both discrete and continuous data. Both will be illustrated in the exam–type questions which follow.

1.4 Exam–style questions

1. Explain briefly each of the following forms of sampling:

(i) simple random sampling.

(ii) stratified random sampling.

(iii) cluster sampling.

2. The table below gives the amounts, in £, spent at a supermarket by a sample of 80 customers. Construct a frequency table for these data, using class intervals $10 \leq x < 20$, $20 \leq x < 30$, $30 \leq x < 40$ and so on. Show frequency, relative frequency and cumulative relative frequency.

40.83	22.10	45.68	39.11	18.92	43.22	37.30	109.97	22.45	42.89
32.95	21.07	41.35	42.54	17.63	19.68	44.62	38.69	66.78	31.99
91.96	25.28	14.82	43.82	44.02	17.31	28.46	28.00	78.28	44.11
29.60	35.34	41.73	65.42	18.65	28.44	45.78	15.99	34.20	35.92
52.03	50.46	58.34	55.03	48.17	43.82	57.77	36.36	53.64	95.75
23.62	34.05	14.47	11.60	23.05	84.30	38.31	41.08	39.47	32.51
32.59	23.58	17.00	25.41	16.78	37.91	80.59	47.32	21.71	28.89
37.97	22.84	47.24	31.34	63.91	81.94	29.06	26.41	36.01	50.78

3. (a) A toy company is to be inspected for the quality and safety of the teddy bears it produces. The inspection team takes a sample of teddy bears from the production line by choosing the first teddy bear at random, and then selecting every 100th teddy bear thereafter. What form of sampling are the team using?

(b) Give one disadvantage of the sampling technique described in part (a).

(c) Another inspection team is to investigate the quality of the company's dolls' houses. In a single working day, the toy company produces 100 houses for Barbie dolls, 200 houses for Cindy dolls and 300 houses for raggy dolls. Suggest a suitable form of sampling to check the quality of dolls' houses produced, and suggest how the inspection team might obtain a sample of size 300.

2 Graphical methods for presenting data

Once we have collected our data, often the best way to summarise this data is through an appropriate graph. Graphs are more eye–catching than tables, and give us an "at–a–glance" picture of our data without too much thought!

2.1 Stem–and–leaf plots

Consider the following data: $11, 12, 9, 15, 21, 25, 19, 8$. The first step is to decide on interval widths – one obvious choice would be to go up in 10s. This would give a *stem unit* of 10 and a *leaf unit* of 1. The stem and leaf plot is constructed as below.

Stem	**Leaf**
0	8 9
1	1 2 5 9
2	1 5

$n = 8$, stem unit $= 10$, leaf unit $= 1$.

Some notes on stem–and–leaf plots

- Always show the stem units and the leaf units.
- The stem unit will usually be either 10 or 1; the corresponding unit for the leaves is usually 1 and 0.1.
- If your sample size is large, you can split each row into two (i.e. 10–14 and 15–19).
- If you have observation written to 2 d.p., always round *down* to obtain values to 1 d.p.

2.2 Bar charts and multiple bar charts

These are a piece of piss to draw. Just be careful with your scale on the y–axis – make sure it covers the entire range of frequencies! And make sure you leave clear gaps between the bars!

2.3 Histograms

Simple frequency histograms can be thought of as "bar charts for continuous data". The only difference is that you need to split the range of your data up into "chunks" or *class intervals*, and you should aim for between 10–15 of these. Once you've done this, simply count the number of observations within each "chunk", and then draw your graph! Remember, unlike bar charts, there are no gaps between the bars in a histogram.

2.4 Exam–style questions

1. Draw a histogram to represent the data given in question 2 of Section 1.4. Comment on the distribution as shown in the histogram. Comment on whether you think that these data come from a Normal distribution and give your reasons.

2. The number of new orders received by a company over the last fourteen working days were recorded as follows:

31 20 25 19 35 36 18 17 68 20 43 21 27 9

Display these data in a stem–and–leaf diagram, and comment.

3 More graphical methods for presenting data

3.1 % relative frequency histograms and polygons

These are just like histograms, but we convert the frequencies into percentages and the heights of the bars now correspond to percentages instead of frequencies. These are useful for illustrating the relative differences between two groups because both are put "onto the same scale".

% relative frequency polygons are constructed in exactly the same way, but now the mid–points of the top of each bar are now connected with straight lines to form a polygon. These are particularly useful for comparing two or more groups (more than two bar charts superimposed could get very messy!).

3.2 Ogives

– Construct a % relative frequency table for your data

– Add a "cumulative" column by adding up the percentages as you go along

– Plot the upper end–point of each "chunk" against the cumulative value

3.3 Pie charts, time series plots and scatterplots

These plots are *all* really easy to construct. You should know how to make appropriate comments on all plots you draw, but there are some things you should look out for with time series plots and scatterplots in particular.

For time series plots, look out for *trend* and *seasonal cycles* in the data. Also look out for any *outliers*.

For scatterplots, is there a *linear association* between the two variables? If so, is this *positive* ("uphill") or *negative* ("downhill")? Is the association *strong*? Or maybe *moderate* or *weak*?

3.4 Exam–style question

The following table contains some data on the weekly household expenditure on food for 200 families.

Interval	Frequency	Interval	Frequency
40 – 60	4	160 – 180	22
60 – 80	12	180 – 200	18
80 – 100	18	200 – 220	6
100 – 120	28	220 – 260	8
120 – 140	48	260 – 280	4
140 – 160	32		

Draw a percentage relative frequency polygon for these data, and comment.

4 Numerical summaries for data

When summarising data numerically, you should use both a measure of *location* and a measure of *spread*. A measure of location is a value which is "typical" of the observations in our sample, and a measure of spread quantifies how "spread out" (or how "fat") our data are.

4.1 Measures of location

1. The mean
The sample mean is given by the formula

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i,$$

which, put more simply, means "add them up and divide by how many you've got".

If your data are given in the form of a frequency table, then the equivalent formula is:

$$\bar{x} = \frac{1}{n}\sum_{j=1}^{k} x_{(j)} f_j,$$

which, put more simply, means "multiply each observation by its frequency, add these bits together and then divide by how many you've got". If you have a grouped frequency table, then you don't know the value of each observation and so just use the midpoint of the group.

2. The median
This is just the observation "in the middle", when the data are put into order. If the sample size (n) is an odd number, we have:

$$\text{median} = \left(\frac{n+1}{2}\right)^{th} \text{ largest observation.}$$

If the sample size (n) is an even number the process is slightly more complicated:

$$\text{median} = \text{average of the } \left(\frac{n}{2}\right)^{th} \text{ and the } \left(\frac{n}{2}+1\right)^{th} \text{ largest observations.}$$

The median is often used if the dataset has an asymmetric profile, since it is not distorted by extreme observations ("outliers").

3. The mode
The mode is simply the most frequently occurring observation. The mode is easy to obtain from a stem–and–leaf plot or a bar–chart. The modal class is easily obtained from a grouped frequency table or a histogram.

4.2 Measures of spread

1. The range

Remember, measures of spread attempt to quantify how "spread out" our data are. The range is arrived at quite intuitively from this definition – it is just the largest value minus the smallest value, which gives the "range" occupied by our data.

2. The inter–quartile range

The IQR measures the range of the middle half of the data, and so is less affected by extreme observations. It is given by $Q3 - Q1$, where

$$Q1 = \frac{(n+1)}{4}\text{th smallest observation}$$
$$Q3 = \frac{3(n+1)}{4}\text{th smallest observation.}$$

3. The variance and standard deviation

The variance can be thought of as "the average squared distance from the mean", and is given by

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2\,.$$

As with the sample mean, we use slightly different formulae if the data are given in the form of a frequency table or a grouped frequency table; however, since most people have a Statistics mode on their calculator (mode `SD` or `Stat`), these formulae are rarely used. *[If you haven't already done so, you should purchase a Casio fX calculator and learn how to use the the Statistics mode; help will be given with this during tutorials in Semester 2.]*

The standard deviation is just the square root of the variance, and is often preferred as it is in the "original units of the data".

4.3 Box and whisker plots

The box and whisker plot graphically displays the lower and upper quartiles, the median, and the maximum and minimum. $Q1$, $Q2$ (the median) and $Q3$ are shown as vertical lines against (usually) a horizontal scale; a these three lines are then made into the "box". A horizontal "whisker" then passes through the middle of the box, extending from the minimum value up to the maximum value.

4.4 Exam–style questions

1. The number of new orders received by a company over the last ten working days were recorded as follows:

$$20 \quad 25 \quad 19 \quad 18 \quad 17 \quad 20 \quad 43 \quad 21 \quad 27 \quad 9$$

Calculate the mean, median, and standard deviation for these data.

2. Estimate the mean household expenditure on food (per week) for the data given in Section 3.4. Why is this just an *estimate* of the mean?

3. The following data are total monthly sales (May 2006, in thousands of pounds) for an Italian pizza chain in ten university towns in England:

117	58	118	120	169	157	202	88	104	149

Draw a box and whisker plot for these data, and comment. What is the inter–quartile range?

5 Introduction to probability

5.1 Definitions

An *experiment* is an activity where we do not know for certain what will happen, but we will *observe* what happens. An *outcome* is one of the possible things that can happen. The *sample space* is the set of all possible outcomes. An *event* is a set of outcomes.

Probabilities are usually expressed in terms of *fractions*, *decimal numbers* or *percentages*.

All probabilities are measured on a scale ranging from zero to one. The probabilities of most events lie strictly between zero and one as an event with probability zero is an *impossible* event and an event with probability one is a *certain* event.

Two events are said to be *mutually exclusive* if both cannot occur simultaneously. Two events are said to be *independent* if the occurrence of one does not affect the probability of the other occurring.

5.2 Measuring probability

1. Classical interpretation
If all possible outcomes are "equally likely" then we can adopt the *classical* approach to measuring probability. For example, if we tossed a fair coin, there are only two possible outcomes – a head or a tail – both of which are equally likely, and hence

$$P(Head) = \frac{1}{2} \quad \text{and} \quad P(Tail) = \frac{1}{2}.$$

In general, calculations follow from the formula

$$P(\text{Event}) = \frac{\text{Total number of outcomes in which event occurs}}{\text{Total number of possible outcomes}}.$$

2. Frequentist interpretation
When the outcomes of an experiment are not equally likely, we can conduct experiments to give us some idea of how likely the different outcomes are. We perform the same experiment a large number of times and observe the outcome. By conducting experiments the probability of an event can easily be estimated using the following formula:

$$P(\text{Event}) = \frac{\text{Number of times an event occurs}}{\text{Total number of times experiment done}}.$$

3. Subjective/Bayesian interpretation
As the name suggests, probabilities within this framework are formulated subjectively using an individual's (sometimes expert) opinion. When we board an aeroplane, we judge the probability of it crashing to be sufficiently small that we are happy to undertake the journey. Similarly, the odds given by bookmakers on a horse race reflect people's beliefs about which horse will win.

5.3 Laws of Probability

Multiplication Law
The probability of two *independent* events A and B both occurring can be written as

$$P(A \text{ and } B) = P(A) \times P(B).$$

Addition Law
The *addition law* describes the probability of any of two or more events occurring. The addition law for two events A and B is

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B).$$

This describes the probability of *either* event A *or* event B happening.

5.4 Exam–style questions

1. In the U.K., 65% of small businesses conduct market research before launching a new product; half of small businesses use a pilot study to check how successful a new product will be before launching it; and 20% of small businesses use *both* market research and a pilot study.

(a) Find the probability that a randomly chosen small business in the U.K. uses *either* market research *or* a pilot study before launching their product.

(b) What percentage of small businesses in the U.K. launch their products without *any* pre–launch research?

2. A chain of clothes retailers has conducted an equal opportunities audit. The number of male and female employees in three types of job – Store Manager, Senior Sales Person and Sales Person – was recorded, the results of which are shown in the table below.

	Male	Female	**Total**
Store Manager	20	4	24
Senior Sales Person	100	140	240
Sales Person	120	202	322
Total	240	346	586

(a) What is the probability that a randomly chosen employee from this survey is male?

(b) What is the probability that a Store Manager is female?

(c) What is the probability that a randomly selected female from this survey is either a Sales Person or a Senior Sales Person?

6 Conditional probability and tree diagrams

6.1 Conditional probability

Consider two events A and B. We write

$$P(B|A)$$

for the probability of B given that A has already happened. We describe $P(B|A)$ as the *conditional probability* of B given A. This gives the following extension to the multiplication rule given previously:

$$P(A \text{ and } B) \quad = \quad P(A) \times P(B|A)$$

6.2 Tree diagrams

Tree diagrams are simple, clear ways of presenting probabilistic information. Suppose we have a biased coin, with $P(\text{Head}) = 0.75$. Then the following tree diagram displays all outcomes, along with their associated probabilities, for two consecutive flips of the coin:

Exam–style question

The probability that a machine needs a new belt within a specified time period is 0.55. The machine could also need a new motor in the same time period – this event occurs with probability 0.3. However, if the belt breaks first, then this probability rises to 0.4.

(a) Find the probability that, within this time period, the machine needs a new belt and then a new motor.

(b) Find the conditional probability that the machine needs a new belt *given that* it needs a new motor.

7 Decision–making using probability

7.1 Expected Monetary Value

The *Expected Monetary Value (EMV)* of a single event is simply the probability of that event multiplied by the monetary value of that outcome. In general,

$$EMV = \sum P(\text{Event}) \times \text{ Monetary value of Event}$$

where the sum is over all possible events. EMV is often combined with decision trees, examples of which are given in the following questions.

7.2 Decision trees

Decision trees are best revised via an example – see the exam–style question below.

7.3 Exam–style question

The manager of a small business has the opportunity to buy a fixed quantity of a new product and offer it for sale for a limited time.

The decision to buy the product and offer it for sale would involve a fixed cost of £150,000. The amount that would be sold is uncertain but the manager's beliefs are expressed as follows.

- There is a probability of 0.3 that sales will be "poor" with an income of £80,000.
- There is a probability of 0.5 that sales will be "medium" with an income of £160,000.
- There is a probability of 0.2 that sales will be "good" with an income of £240,000.

For an additional fixed cost of £20,000, the product can be sold for a trial period before a final decision is made. No income is made from this trial. The result of the trial will be "poor" with probability 0.33, "medium" with probability 0.40 or "good" with probability 0.27. Knowing the outcome of the trial changes the probabilities for the main sales project:

	Main sales probabilities		
Trial outcome	Poor	Medium	Good
Poor	0.7	0.2	0.1
Medium	0.2	0.6	0.2
Good	0.1	0.2	0.7

The manager will make decisions on the basis of expected monetary value.

(a) Draw a decision tree for this problem.

(b) Find the expected monetary value of a decision to go ahead with the product without a trial.

(c) Complete the solution of the decision problem and determine the optimal course of action for the company.

8 Discrete probability models

8.1 Permutations and combinations

The number of ways of choosing r objects out of n, *when the ordering matters*, is given by

$$^n\mathrm{P}_r = \frac{n!}{(n-r)!},$$

where

$$n! = n(n-1)(n-2)(n-3) \times \cdots \times 3 \times 2 \times 1.$$

The example to think about here is the stolen credit card example discussed in lecture 8.

sometimes the ordering of the chosen numbers is not important – all that's important is that we get the correct numbers! If this is the case, the number of ways of choosing r objects out on n is given by

$$^n\mathrm{C}_r = \frac{n!}{r!(n-r)!}.$$

An example of where ordering is not important is the national lottery – we don't care what order the numbers come out in, as long as our numbers are there! For the jackpot, the number of ways of choosing 6 correct numbers from all 49 is given by $^{49}\mathrm{C}_6$.

8.2 Probability distributions

The *probability distribution* of a discrete random variable X is the list of all possible values X can take and the probabilities associated with them. For example, if the random variable X is the outcome of a roll of a die then the probability distribution for X is:

x	$P(X = x)$
1	1/6
2	1/6
3	1/6
4	1/6
5	1/6
6	1/6
sum	1

8.3 The binomial distribution

Suppose the following statements hold:

- There are a fixed number of trials (n);
- There are only two possible outcomes for each trial ('success' or 'failure');
- There is a constant probability of 'success', p;
- The outcome of each trial is independent of any other trial.

Then the number of successes, X, follows a **binomial distribution**.

We write $X \sim \mathrm{bin}(n, p)$, and

$$P(X = r) \quad = \quad {}^{n}\mathrm{C}_{r} p^{r} (1-p)^{n-r}, \qquad x = 0, 1, \ldots, n.$$

If $X \sim \mathrm{bin}(n, p)$, then

$$\begin{aligned} \mathrm{E}(X) &= n \times p \qquad \text{and} \\ \mathrm{Var}(X) &= n \times p \times (1-p). \end{aligned}$$

8.4 The Poisson distribution

Suppose the following hold:

- There is no natural upper limit to the number of trials;
- Events occur independently, at a constant rate (λ).

Then the number of events, X, occurring with rate λ, has a **Poisson distribution**.

We write $X \sim \mathrm{Po}(\lambda)$, and

$$P(X = r) \quad = \quad \frac{e^{-\lambda} \times \lambda^{r}}{r!}, \qquad x = 0, 1, \ldots$$

If $X \sim \mathrm{Po}(\lambda)$, then

$$\begin{aligned} \mathrm{E}(X) &= \lambda \qquad \text{and} \\ \mathrm{Var}(X) &= \lambda. \end{aligned}$$

8.5 Exam–style questions

1. A new Mercedes–Benz car franchise forecasts that it will sell around three of its most expensive models each day.

(a) What probability distribution might be reasonable to use to model the number of cars sold each day?

(b) What is the expected number and standard deviation of the number of cars sold each day?

(c) What is the probability that no cars are sold on a particular day?

2. An operator at a call centre has 20 calls to make in an hour. History suggests that they will be answered 55% of the time. Let X be the number of answered calls in an hour.

(a) What probability distribution does X have?

(b) What is the mean and standard deviation of X?

(c) Calculate the probability of getting a response exactly 9 times.

(d) Calculate the probability of getting less than 2 responses.

9 The Normal distribution

The *Normal* distribution is possibly the best–known and most–used continuous probability distribution. The Normal distribution has two parameters: the mean, μ, and the standard deviation, σ. Its probability density function (pdf) has a "bell shaped" profile:

If a random variable X has a Normal distribution with mean μ and variance σ^2, then we write

$$X \sim N\left(\mu, \sigma^2\right).$$

The *standard Normal distribution*, usually denoted by Z, has zero mean and a variance of 1, and we have tables of probabilities for this particular Normal distribution. Any Normally distributed random variable X can be transformed into the *standard* Normal distribution using the formula:

$$Z \quad = \quad \frac{X - \mu}{\sigma};$$

once we've performed this "slide–squash" transformation, we obtain probabilities by reference to statistical tables. Don't forget, tables give "less than" probabilities. So, for a "greater than" probability, subtract the "less than" probability found in the tables from 1. Similarly, we can use tables to find the probability that our random variable lies between two values.

Exam–style question

A call centre's declared policy is to respond to calls within 50 seconds. In reality they respond according to a Normal distribution with a mean of 60 seconds and a standard deviation of 10 seconds.

(a) What proportion of calls receive a late response?

(b) What proportion of calls are answered between 50 and 70 seconds?

(c) To how many seconds would the policy have to be adjusted, for only 5% of calls to be answered late?

10 More continuous probability distributions

10.1 The Uniform distribution

The *uniform distribution* is the most simple continuous distribution. As the name suggests, it describes a variable for which all possible outcomes are equally likely. If the random variable X follows a Uniform distribution, we write

$$X \sim U(a, b).$$

Probabilities can be calculated using the formula

$$P(X \leq x) = \begin{cases} 0 & \text{for } x < a \\ \dfrac{x-a}{b-a} & \text{for } a \leq x \leq b \\ 1 & \text{for } x > b, \end{cases}$$

and the mean and variance are given by

$$\mathrm{E}(X) = \frac{a+b}{2}, \qquad \mathrm{Var}(X) = \frac{(b-a)^2}{12}.$$

10.2 The exponential distribution

The *exponential distribution* is another common distribution that is used to describe continuous random variables. It is often used to model lifetimes of products and times between "random" events such as arrivals of customers in a queueing system or arrivals of orders. The distribution has one parameter, λ. If our random variable X follows an exponential distribution, then we say

$$X \sim \exp(\lambda).$$

Probabilities can be calculated using

$$P(X \leq x) = \begin{cases} 0 & \text{for } x < 0 \\ 1 - e^{-\lambda x} & \text{for } x > 0, \end{cases}$$

and the mean and variance are given by

$$\mathrm{E}(X) = \frac{1}{\lambda}, \qquad \mathrm{Var}(X) = \frac{1}{\lambda^2}.$$

10.3 Exam–style questions

1. A local authority is responsible for a stretch of road 3km long through a town. A gas main runs along the length of the road. The gas company has requested permission to dig up the road in one place but has neglected to tell the authority exactly where.

(a) Let Y be the distance of the gas works from one end of the road. Sketch the pdf of Y.

(b) What distribution does Y take?

(c) The stretch of road between 1.5 and 2.75 kilometres from one end goes through the town centre and gas works there would cause severe disruption. What is the probability that this happens?

2. The time (in minutes) between uses of a vending machine is modelled as an exponential distribution with rate 0.2.

(a) What is the mean time between uses?

(b) What is the probability that there is a gap of more than 10 minutes between uses?