

# Chapter 7

## Decision-making using probability

In this chapter, we look at how we can use probability in order to aid decision-making.

### 7.1 Expected Monetary Value

Intuition should now help to explain how probability can be used to aid the decision-making process. For example, suppose we're considering launching a new product on the market. We conduct a pre-launch questionnaire and 86 out of the 100 questionnaire respondents say that they *would* buy our product if it was on the market. We might translate this into the following probability statement:

$$P(\text{product successful}) = \frac{86}{100} = 0.86,$$

which is quite good, and so surely we should launch the product? It looks promising! But... we should also consider the *financial* outcome of our situation. For example, if the product is successful, we might make a reasonable profit, but if the product is *not* successful, we could stand to lose a lot more than we would gain under success (set-up costs, advertising, production costs etc), and such financial considerations could outweigh the high probability of success alone. So, in real-life scenarios, not only do we use probability to aid the decision-making process, but also the financial implications. This is achieved by weighting the probability of different outcomes by their value, which is often financial. The *Expected Monetary Value* (*EMV*) of a single event is simply the probability of that event multiplied by the monetary value of that outcome.

#### Example 1

If you would win £5 if you pulled an ace from a pack of cards, the *EMV* would be

$$EMV(Ace) = \frac{4}{52} \times 5 = 0.38.$$

In other words, if you repeated this bet a large number of times, overall you would come out, on average, 38 pence better off per bet. Therefore you would want to pay no more than 38p for such a bet.

**Example 2**

Consider another bet. When rolling a die, if it's a six you have to pay £5 but if it's any other number you receive £2.50. Would you take on this bet?

Probability	Financial outcome
$P(6) = 1/6$	−£5
$P(\text{Not a } 6) = 5/6$	£2.50

Therefore

$$EMV(\text{Six}) = \frac{1}{6} \times -5.00 = -0.833$$

$$EMV(\text{Not a Six}) = \frac{5}{6} \times 2.50 = 2.0833$$

and hence the expected monetary value of the bet is

$$EMV(\text{Bet}) = -0.833 + 2.083 = 1.25.$$

Therefore, in the long run, this would be a bet to take on as it has a positive expected monetary value.

**Example 3: The National Lottery**

In a recent lottery draw, the prizes were

Number of balls matched	Probability	Prize
6	0.000000071	£2.4M
5 plus bonus	0.000000429	£240K
5	0.000018449	£3K
4	0.000968619	£100
3	0.0177	£10
< 3	0	£0

(We will see how to work out these probabilities next week.)

The *EMV* of the bet is

$$\begin{aligned}
 EMV &= P(\text{match 6 balls}) \times \text{Prize}(\text{match 6 balls}) \\
 &\quad + P(\text{match 5 balls plus bonus}) \times \text{Prize}(\text{match 5 balls plus bonus}) \\
 &\quad + P(\text{match 5 balls}) \times \text{Prize}(\text{match 5 balls}) \\
 &\quad + P(\text{match 4 balls}) \times \text{Prize}(\text{match 4}) \\
 &\quad + P(\text{match 3 balls}) \times \text{Prize}(\text{match 3 balls}) \\
 &= 2.4M \times 0.000000071 + \dots \\
 &= 0.6176.
 \end{aligned}$$

Therefore, a fair price for a ticket in this particular lottery is around 62p. The difference between this and the standard £1 charge for a ticket goes to “good causes” and, of course, *Camelot’s profits*.

In general, the expected monetary value of a project or bet is given by the formula

$$EMV = \sum P(\text{Event}) \times \text{Monetary value of Event}$$

where the sum is over all possible events. The *EMV* of a project can be used as a decision criterion for choosing between different projects and has applications in a large number of situations. This is illustrated by the following example.

#### Example 4

A small company is trying to decide how to launch a new and innovative product. It could go for a direct approach, launching onto the whole of the domestic market through traditional distribution channels, or it could launch only on the internet. A third option exists where the product is licensed to a larger company through the payment of a licence fee irrespective of the success of the product. How should the company launch the product? The company has done some initial market research and the managing director believes the probability of the product being successful can be classed into three categories: high, medium or low. She thinks that these categories will occur with probabilities 0.2, 0.35 and 0.45 respectively and her thoughts on the likely profits (in £K) to be earned in each plan are

	High	Medium	Low
Direct	100	55	-25
Internet	46	25	15
Licence	20	20	20

The *EMV* of each plan can be calculated as follows:

$$\begin{aligned}
 EMV(\text{Direct}) &= 0.2 \times 100 + 0.35 \times 55 + 0.45 \times (-25) = \text{£}28\text{K} \\
 EMV(\text{Internet}) &= 0.2 \times 46 + 0.35 \times 25 + 0.45 \times 15 = \text{£}24.7\text{K} \\
 EMV(\text{Licence}) &= 0.2 \times 20 + 0.35 \times 20 + 0.45 \times 20 = \text{£}20\text{K}.
 \end{aligned}$$

On the basis of expected monetary value, the best choice is the Direct approach.

## 7.2 Decision trees

In the last example we had to make a decision. When we include a decision in a tree diagram (see last week) we use a rectangular node, called a *decision node* to represent the decision. The diagram is then called a *decision tree*. There are no probabilities at a decision node but we evaluate the expected monetary values of the options. In a decision tree the first node is always a decision node. There may also be other decision nodes. If there is another decision node then we evaluate the options there and choose the best and the expected value of this option becomes the expected value of the branch leading to the decision node.

The decision tree for the last example would look like this:

Now consider a more complicated example.

The manager of a small I.T. sales company has the opportunity to buy a fixed quantity of a new type of soundcard for home PCs which they can then offer for sale to clients. The decision to buy the product and offer it for sale would involve a fixed cost of £200,000. The number of soundcards that would be sold is uncertain, but the manager's prior beliefs are expressed as follows.

- Sales will be “poor” with probability 0.2; this will result in an income of £100,000.
- Sales will be “moderate” with probability 0.5; this will result in an income of £220,000.
- Sales will be “good” with probability 0.3; this will result in an income of £350,000.

For an additional fixed cost of £30,000, market research can be conducted to aid the decision-making process. The outcome of the market research can be either positive or negative, with probabilities 0.58 and 0.42 respectively. Knowing the outcome of the market research changes the probabilities for the main sales project as follows:

Market research	Main sales probabilities		
	Poor	Moderate	Good
Positive	0.15	0.45	0.4
Negative	0.6	0.35	0.05

The manager will make decisions based on expected monetary value.

- (a) Draw a decision tree for this problem.
- (b) Use expected monetary value to determine the optimal course of action for the company.

*This page is left blank for your solution to the last question*

### 7.3 Exercises

1. A company has developed a new manufacturing process which they believe will revolutionise their industry. They are, however, uncertain how they should go about exploiting this advance.

Initial indications of the likely success of marketing the process are 55%, 30% 15% for “high success”, “medium success” and “probable failure” (respectively). The company has three options; they can go ahead and develop the technology themselves, license it or sell the rights to it. The financial outcomes (in £ millions) for each option are given in the table below.

	“high success”	“medium success”	“failure”
Develop	80	40	−100
Licence	40	30	0
Sell	25	25	25

- (a) Draw a decision tree to represent the company’s problem.
  - (b) Calculate the *Expected Monetary Value* for all possible decisions the company may take and hence determine the optimal decision for the company.
2. The owner of a small business has the right to have a retail stall at a large festival to be held during the summer. She judges that this would either be a success or a failure and that the probability that it is a success is 0.4. If the stall was a success, the net income from it would be £90,000. If the stall was a failure, there would be a net loss of £30,000 from it. To help to make the decision, the owner could pay for market research. This would cost £5,000. The market research will either give a positive indication or a negative indication. The conditional probability that it gives a positive indication, given that the stall will actually be a success, is 0.75. The conditional probability that it gives a positive indication, given that the stall will actually be a failure, is  $1/3$ .

The owner has various options:

- Do nothing.
- Go ahead without market research.
- Pay for the market research.
- Sell her right to a stall for £10,000.

If she pays for the market research then, depending on the outcome, she can:

- Do nothing more.
- Go ahead.
- Sell her right to a stall. If the market research gave a positive indication the price would be £35,000. If the market research gave a negative indication the price would be only £3,000.

What should she do?

This is a fairly complicated question so it is best to tackle it in stages.

- (a) using a probability tree for the success or failure of the stall and the market research outcome, find the following probabilities.
  - i. The probability of a positive market research outcome.
  - ii. The probability of a negative market research outcome.
  - iii. The conditional probability of a successful stall given a positive market research outcome.
  - iv. The conditional probability of a failure given a positive market research outcome.
  - v. The conditional probability of a successful stall given a negative market research outcome.
  - vi. The conditional probability of a failure given a negative market research outcome.
- (b) Represent the owner's decision problem using a tree diagram.
- (c) Suppose that the owner has the market research done and that the outcome is positive. Evaluate the expected monetary values, under these circumstances, of the three options:
  - Sell.
  - Go ahead.
  - Do nothing more.and hence find what she should do if these circumstances arise.
- (d) Suppose that the owner has the market research done and that the outcome is negative. Evaluate the expected monetary values, under these circumstances, of the three options:
  - Sell.
  - Go ahead.
  - Do nothing more.and hence find what she should do if these circumstances arise.
- (e) Hence find the expected monetary value of the initial option of "Pay for the market research."
- (f) Find the expected monetary values of the three other initial options:
  - Do nothing.
  - Go ahead without market research.
  - Sell her right to a stall for £10,000.
- (g) Determine the owner's best strategy.