Differentiate each of the following.

1. $y = x^6$.

2. $y = x^2 + 2x^3$.

3. $y = \ln x + \sqrt{x}$.

4. $T = 13q^2 + 7q$.

5. $y = 6x^{\frac{3}{2}} - \frac{1}{\sqrt{x}}$.

6. $y = \frac{64}{3x^3}$.

7. $A = \frac{p^6}{4} - 4p + \frac{3}{p^2}$.

8. The curve C is given by the function $f(x)$ and passes through the point $P(2, 5)$. We know that, for C,

$$f'(x) = 9x^2 - 12x + 2.$$

   (a) Find $f(x)$.

   (b) Verify that the point $Q(5, 241)$ lies on C.

   (c) Find the gradient of C at the point $x = 3$.

   (d) Find the equation of the tangent to C at $x = 3$ in the form $y = mx + c$.

   (e) Find the equation of the normal at P.

9. The curve S is given by the function $f(x)$ and passes through the point $P(2, 0)$. We know that, for S,

$$f'(x) \;=\; 4x - 1.$$

   (a) Find $f(x)$.

   (b) Find the co–ordinates of the other point Q for which S intersects the x–axis.

   (c) Calculate the gradient of S at P and Q.

   (d) Find the co–ordinates of the point R for which the gradient is zero. Hence, using information about the points P, Q and R, sketch S.

   (e) Find the equation of the tangent to S at P in the form $y = mx + c$.

10. The curve C is given by

$$f(x) \;=\; x^3 + \frac{3}{x} + 2\sqrt{x} + 2.$$

   (a) Find the gradient of C at the point P, where $x = 4$.

   (b) Find the equation of the tangent to C at P in the form $y = mx + c$.

11. The curve passing through $P(-3, 0)$ and given by the equation $y = (x + A)(Ax + B)$ has

$$\frac{dy}{dx} = 2(3x + 5).$$

   (a) Find A and B.

   (b) Find the x co–ordinate of the point Q that intersects the x–axis.

   (c) Find the equation of the tangent to y at Q in the form $y = mx + c$.