

Statistical formulae and tables

1 Sample statistics

Lower quartile: $Q_1 = \frac{(n+1)}{4}$ th smallest observation.

Median: $Q_2 = \frac{(n+1)}{2}$ th smallest observation.

Upper quartile: $Q_3 = \frac{3(n+1)}{4}$ th smallest observation.

Sample mean:
$$\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i$$

Sample variance:
$$s^2 = \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2 = \frac{1}{n-1} \left\{ \sum_{i=1}^n x_i^2 - n\bar{x}^2 \right\}$$

2 Combining probabilities

Addition

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

If A and B are mutually exclusive then

$$P(A \text{ or } B) = P(A) + P(B)$$

Multiplication

$$P(A \text{ and } B) = P(A)P(B | A)$$

If A and B are independent then

$$P(A \text{ and } B) = P(A)P(B)$$

3 Probability distributions

Discrete distributions	$P(X = r)$	Mean	Variance
Binomial, $X \sim \text{Bin}(n, p)$	${}^nC_r p^r (1-p)^{n-r}$	np	$np(1-p)$
Poisson, $X \sim \text{Po}(\lambda)$	$\frac{e^{-\lambda} \lambda^r}{r!}$	λ	λ
Continuous distributions	$P(X \leq x)$	Mean	Variance
Uniform, $X \sim U(a, b)$	$\frac{x-a}{b-a}$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$
Exponential, $X \sim \text{Exp}(\lambda)$	$1 - e^{-\lambda x}$	$1/\lambda$	$1/\lambda^2$
Normal, $X \sim N(\mu, \sigma^2)$	$P\left(Z \leq \frac{X-\mu}{\sigma}\right)$, where $Z \sim N(0, 1)$	μ	σ^2

4 Confidence intervals and hypothesis tests

Confidence interval for the population mean μ

σ known: $\bar{x} \pm z \times \sqrt{\sigma^2/n}$;

σ unknown: $\bar{x} \pm t \times \sqrt{s^2/n}$,

where z is a critical value from the standard Normal distribution (i.e. $z = 1.645, 1.96$ and 2.576 for a 90%, 95% and 99% interval (respectively), and t is a critical value from the t distribution with $\nu = n - 1$ degrees of freedom.

Hypothesis test	Test statistic	Other comments
<i>Tests for one mean</i>		
σ known	$z = \frac{ \bar{x} - \mu }{\sqrt{\sigma^2/n}}$	
σ unknown	$t = \frac{ \bar{x} - \mu }{\sqrt{s^2/n}}$	$\nu = n - 1$
<i>Tests for two means</i>		
σ_1, σ_2 known	$z = \frac{ \bar{x}_1 - \bar{x}_2 }{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$	
σ_1, σ_2 unknown	$t = \frac{ \bar{x}_1 - \bar{x}_2 }{s \times \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$	$s = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1 + n_2 - 2}}$ $\nu = n_1 + n_2 - 2$
<i>Goodness-of-fit tests</i>	$X^2 = \sum \frac{(O-E)^2}{E}$	$\nu = (\text{no. of categories after pooling}) - (\text{no. of estimated parameters}) - 1$
<i>Tests for independence</i>	$X^2 = \sum \frac{(O-E)^2}{E}$	$E = \frac{\text{row total} \times \text{column total}}{\text{overall total}}$ $\nu = (\text{no. of rows} - 1) \times (\text{no. of columns} - 1)$

5 Correlation and linear regression

The sample correlation coefficient is given by

$$r = \frac{S_{XY}}{\sqrt{S_{XX} \times S_{YY}}},$$

where

$$\begin{aligned} S_{XY} &= \left(\sum xy \right) - n\bar{x}\bar{y}, \\ S_{XX} &= \left(\sum x^2 \right) - n\bar{x}^2, \quad \text{and} \\ S_{YY} &= \left(\sum y^2 \right) - n\bar{y}^2. \end{aligned}$$

The simple linear regression equation is given by

$$Y = \alpha + \beta X + \epsilon,$$

where $\{\epsilon_i\}$ are independent $N(0, \sigma^2)$ random variables and α and β can be estimated using

$$\begin{aligned} \hat{\beta} &= \frac{S_{XY}}{S_{XX}} \quad \text{and} \\ \hat{\alpha} &= \bar{y} - \hat{\beta}\bar{x}. \end{aligned}$$

6 Statistical tables

	One-tailed test Two-tailed test	p				
		10% 20%	5% 10%	2.5% 5%	1% 2%	0.5% 1%
ν	1	3.078	6.314	12.706	31.821	63.657
	2	1.886	2.920	4.303	6.965	9.925
	3	1.638	2.353	3.182	4.541	5.841
	4	1.533	2.132	2.776	3.747	4.604
	5	1.476	2.015	2.571	3.365	4.032
	6	1.440	1.943	2.447	3.143	3.707
	7	1.415	1.895	2.365	2.998	3.449
	8	1.397	1.860	2.306	2.896	3.355
	9	1.383	1.833	2.262	2.821	3.250
	10	1.372	1.812	2.228	2.764	3.169
	11	1.363	1.796	2.201	2.718	3.106
	12	1.356	1.782	2.179	2.681	3.055
	13	1.350	1.771	2.160	2.650	3.012
	14	1.345	1.761	2.145	2.624	2.977
	15	1.341	1.753	2.131	2.602	2.947
	16	1.337	1.746	2.120	2.583	2.921
	17	1.333	1.740	2.110	2.567	2.898
	18	1.330	1.734	2.101	2.552	2.878
	19	1.328	1.729	2.093	2.539	2.861
	20	1.325	1.725	2.086	2.528	2.845
	21	1.323	1.721	2.080	2.518	2.831
	22	1.321	1.717	2.074	2.508	2.819
	23	1.319	1.714	2.069	2.500	2.807
	24	1.318	1.711	2.064	2.492	2.797
	25	1.316	1.708	2.060	2.485	2.787
	26	1.315	1.706	2.056	2.479	2.779
	27	1.314	1.703	2.052	2.473	2.771
	28	1.313	1.701	2.048	2.467	2.763
	29	1.311	1.699	2.045	2.462	2.756
	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots
	∞	1.282	1.645	1.960	2.326	2.576

Table 1: This table contains values of t for which $\Pr(T > t) = p$, where $T \sim t_\nu$. In a two-tailed test, the tabulated values correspond to $\Pr(|T| > t) = p$

		p				
		50%	10%	5%	1%	0.1%
ν	1	0.45	2.17	3.84	6.63	10.83
	2	1.39	4.61	5.99	9.21	13.82
	3	2.37	6.25	7.82	11.34	16.27
	4	3.36	7.78	9.49	13.28	18.47
	5	4.34	9.24	11.07	15.09	20.52
	6	5.35	10.64	12.59	16.81	22.46
	7	6.35	12.02	14.07	18.48	24.32
	8	7.34	13.36	15.51	20.09	26.13
	9	8.34	14.68	16.92	21.67	27.88
	10	9.34	15.99	18.31	23.21	29.59
	12	11.34	18.55	21.03	26.22	32.91
	15	14.34	22.31	25.00	30.58	37.70
	20	19.34	28.41	31.41	37.57	45.32
	25	24.34	34.38	37.65	44.31	52.62
	30	29.34	40.26	43.77	50.89	59.70

Table 2: This table contains values of x for which $\Pr(X^2 > x) = p$, where $X^2 \sim \chi_\nu^2$

z	-0.09	-0.08	-0.07	-0.06	-0.05	-0.04	-0.03	-0.02	-0.01	0.00
-2.9	0.0014	0.0014	0.0015	0.0015	0.0016	0.0016	0.0017	0.0018	0.0018	0.0019
-2.8	0.0019	0.0020	0.0021	0.0021	0.0022	0.0023	0.0023	0.0024	0.0025	0.0026
-2.7	0.0026	0.0027	0.0028	0.0029	0.0030	0.0031	0.0032	0.0033	0.0034	0.0035
-2.6	0.0036	0.0037	0.0038	0.0039	0.0040	0.0041	0.0043	0.0044	0.0045	0.0047
-2.5	0.0048	0.0049	0.0051	0.0052	0.0054	0.0055	0.0057	0.0059	0.0060	0.0062
-2.4	0.0064	0.0066	0.0068	0.0069	0.0071	0.0073	0.0075	0.0078	0.0080	0.0082
-2.3	0.0084	0.0087	0.0089	0.0091	0.0094	0.0096	0.0099	0.0102	0.0104	0.0107
-2.2	0.0110	0.0113	0.0116	0.0119	0.0122	0.0125	0.0129	0.0132	0.0136	0.0139
-2.1	0.0143	0.0146	0.0150	0.0154	0.0158	0.0162	0.0166	0.0170	0.0174	0.0179
-2.0	0.0183	0.0188	0.0192	0.0197	0.0202	0.0207	0.0212	0.0217	0.0222	0.0228
-1.9	0.0233	0.0239	0.0244	0.0250	0.0256	0.0262	0.0268	0.0274	0.0281	0.0287
-1.8	0.0294	0.0301	0.0307	0.0314	0.0322	0.0329	0.0336	0.0344	0.0351	0.0359
-1.7	0.0367	0.0375	0.0384	0.0392	0.0401	0.0409	0.0418	0.0427	0.0436	0.0446
-1.6	0.0455	0.0465	0.0475	0.0485	0.0495	0.0505	0.0516	0.0526	0.0537	0.0548
-1.5	0.0559	0.0571	0.0582	0.0594	0.0606	0.0618	0.0630	0.0643	0.0655	0.0668
-1.4	0.0681	0.0694	0.0708	0.0721	0.0735	0.0749	0.0764	0.0778	0.0793	0.0808
-1.3	0.0823	0.0838	0.0853	0.0869	0.0885	0.0901	0.0918	0.0934	0.0951	0.0968
-1.2	0.0985	0.1003	0.1020	0.1038	0.1056	0.1075	0.1093	0.1112	0.1131	0.1151
-1.1	0.1170	0.1190	0.1210	0.1230	0.1251	0.1271	0.1292	0.1314	0.1335	0.1357
-1.0	0.1379	0.1401	0.1423	0.1446	0.1469	0.1492	0.1515	0.1539	0.1562	0.1587
-0.9	0.1611	0.1635	0.1660	0.1685	0.1711	0.1736	0.1762	0.1788	0.1814	0.1841
-0.8	0.1867	0.1894	0.1922	0.1949	0.1977	0.2005	0.2033	0.2061	0.2090	0.2119
-0.7	0.2148	0.2177	0.2206	0.2236	0.2266	0.2296	0.2327	0.2358	0.2389	0.2420
-0.6	0.2451	0.2483	0.2514	0.2546	0.2578	0.2611	0.2643	0.2676	0.2709	0.2743
-0.5	0.2776	0.2810	0.2843	0.2877	0.2912	0.2946	0.2981	0.3015	0.3050	0.3085
-0.4	0.3121	0.3156	0.3192	0.3228	0.3264	0.3300	0.3336	0.3372	0.3409	0.3446
-0.3	0.3483	0.3520	0.3557	0.3594	0.3632	0.3669	0.3707	0.3745	0.3783	0.3821
-0.2	0.3859	0.3897	0.3936	0.3974	0.4013	0.4052	0.4090	0.4129	0.4168	0.4207
-0.1	0.4247	0.4286	0.4325	0.4364	0.4404	0.4443	0.4483	0.4522	0.4562	0.4602
0.0	0.4641	0.4681	0.4721	0.4761	0.4801	0.4840	0.4880	0.4920	0.4960	0.5000

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986

Table 3: This table contains values of $\Pr(Z < z)$, where $Z \sim N(0, 1)$