Bayesian Inference for Clustered Extremes

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Structure of this talk

1. **Motivation and background**

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2. **Inference for the extremal index**
   - Review of existing methods
   - Limitations/difficulties
   - A Bayesian sampling scheme

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2. **Inference for the extremal index**
   - Review of existing methods
   - Limitations/difficulties
   - A Bayesian sampling scheme
3. **Implementation**
   - Simulated data – just to check!
   - Extreme wind speeds observed at High Bradfield

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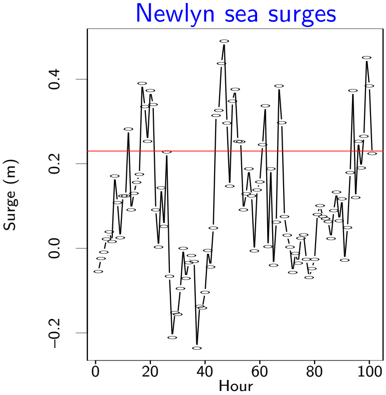
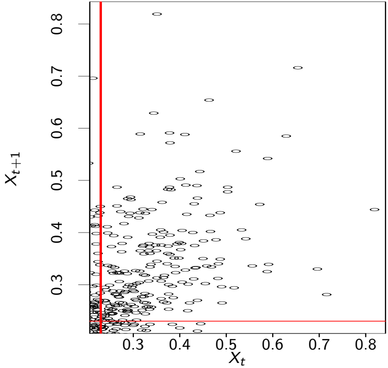
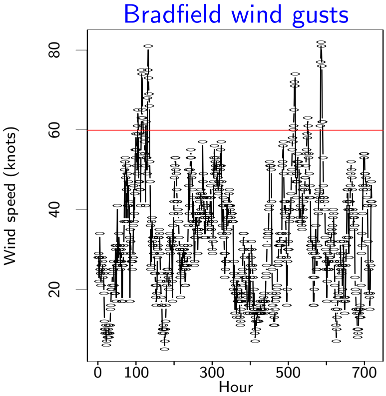
Being able to quantify this extremal dependence, and any other storm characteristics induced by this, can be of interest to meteorologists and/or engineers.

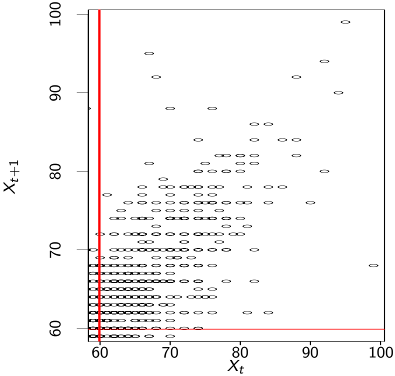
Examples


Newlyn sea surges
Surge (m)
Hour
0.4
0.2
0.0
−0.2
0
20
40
60
80
100


X_{t+1}
X_t
0.8
0.7
0.6
0.5
0.4
0.3
0.3
0.4
0.5
0.6
0.7
0.8

Examples


Bradfield wind gusts
Wind speed (knots)
20
40
60
80
0
100
300
500
700
Hour


X_{t+1}
60
70
80
90
100
60
70
80
90
100
X_t

The extremal index

Let $\tilde{X}_1, \tilde{X}_2, \ldots, \tilde{X}_n$ be the first n observations of a stationary series satisfying Leadbetter's $D(u_n)$ condition (Leadbetter *et al.*, 1983), and let $\tilde{M}_n = \max\{\tilde{X}_1, \tilde{X}_2, \ldots \tilde{X}_n\}$.

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$$\Pr\left\{\left(\tilde{M}_n - b_n\right)/a_n \leq x\right\} \;\to\; G^{\theta}(x) \qquad (1)$$

for some $0 \leq \theta \leq 1$ (Leadbetter *et al.*, 1983).

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The parameter θ is known as the **extremal index** and is a key parameter which quantifies the extent of extremal clustering.

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Thus, identifying **clusters** can be important in estimating θ.

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as well as a hybrid of two of these in a Bayesian setting.

We will also consider the viability of these approaches for estimating other cluster characteristics, often referred to as **cluster functionals**.

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- The margins are of **Gumbel** form, i.e. $F(x) = \exp\{\exp(-x)\}$

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Thus, if we can simulate successive values from the logistic model in (2), then we can compare methods for estimating θ.

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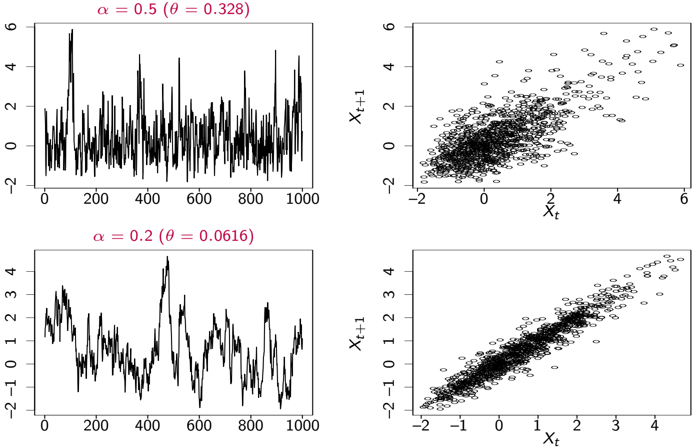

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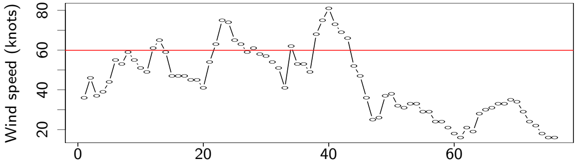
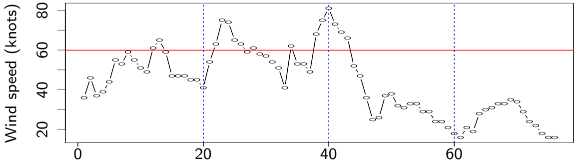
Runs declustering (separation interval $= \kappa$)



Blocks declustering (block length $= \tau$)



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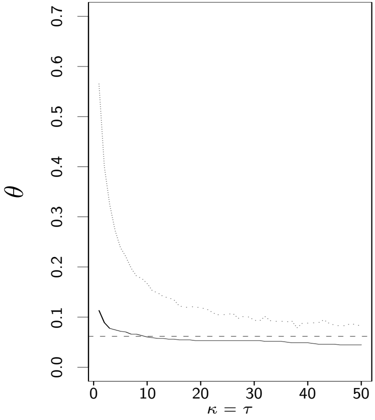
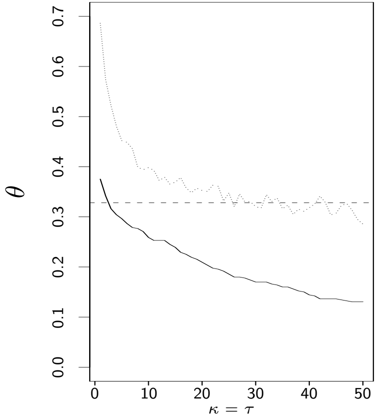
- Other cluster functionals also sensitive to the choice of κ/τ!

Cluster size methods





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$$G(x;\mu,\sigma,\xi) \quad = \quad \exp\left\{-\left[1+\xi\left(\frac{x-\mu}{\sigma}\right)\right]_{+}^{-1/\xi}\right\}.$$

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Ancona–Navarrete and Tawn (2000) suggest simultaneous estimation of the parameter vector $(\mu, \sigma, \xi, \theta)$ by treating components of the vector $(\mathbf{M}_\tau, \tilde{\mathbf{M}}_\tau)$ as independent GEV random variables.

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- No clusters are identified! And so we cannot estimate cluster characteristics that are not direct functions of θ

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- Assume a first–order Markov structure for extremes
- Model the distribution of consecutive pairs using a bivariate extreme value distribution (such as the logistic model), estimating an appropriate dependence parameter (such as α)
- Repeatedly simulate clusters of extremes using the fitted model, and observe what happens!

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- Is a first–order Markov assumption appropriate?
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- Is a first–order Markov assumption appropriate?
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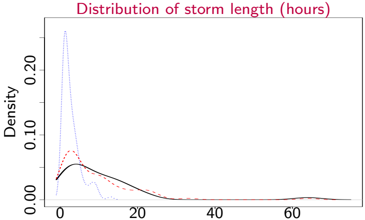
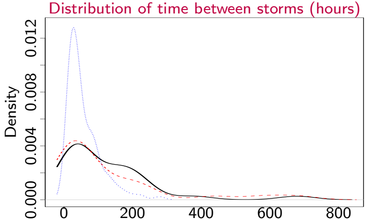
- How can we check the above two points? Ad–hoc procedures?

Simulation methods





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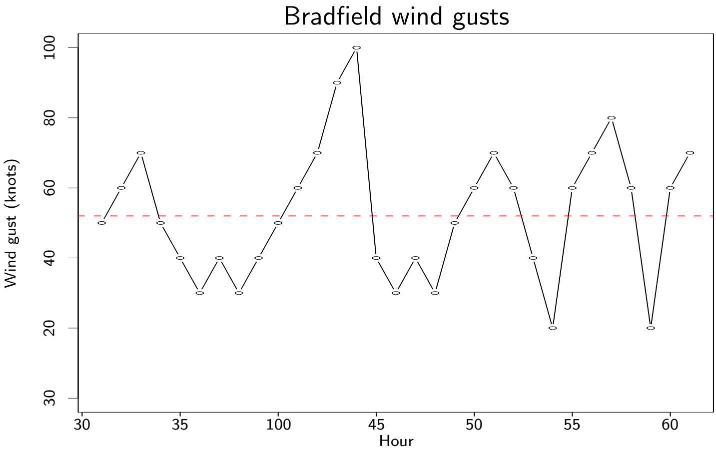
- No need to specify an arbitrary value for κ now – let this be governed by the level of extremal dependence in the process (via θ);
- What about θ? Likelihood based on inter–arrival times performs poorly, so they use a non–parametric approach.

Automatic declustering (Ferro and Segers, 2003)



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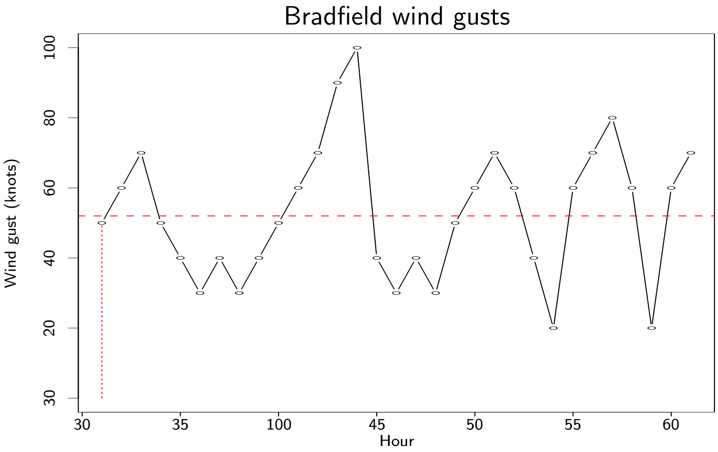

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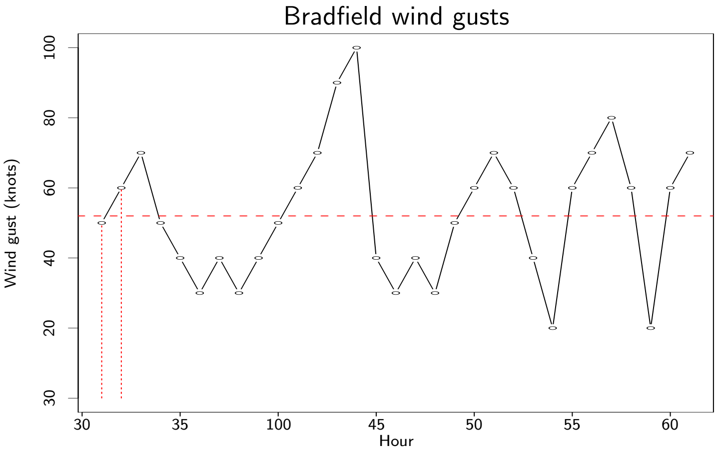

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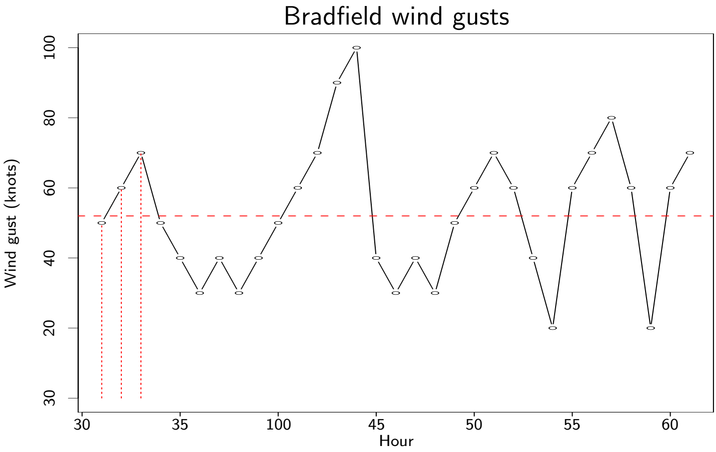

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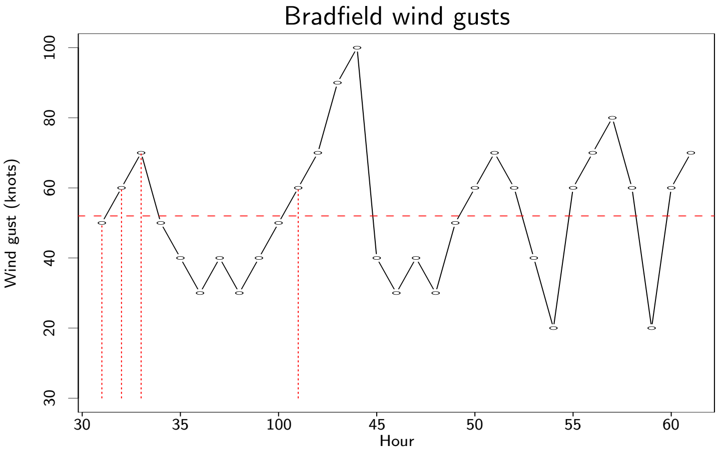

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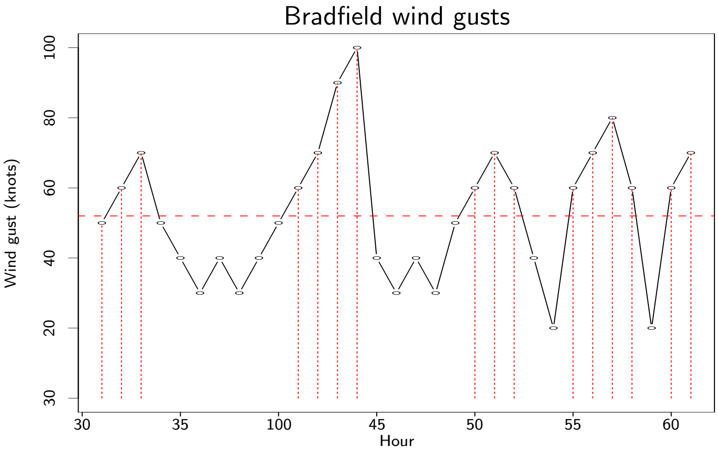

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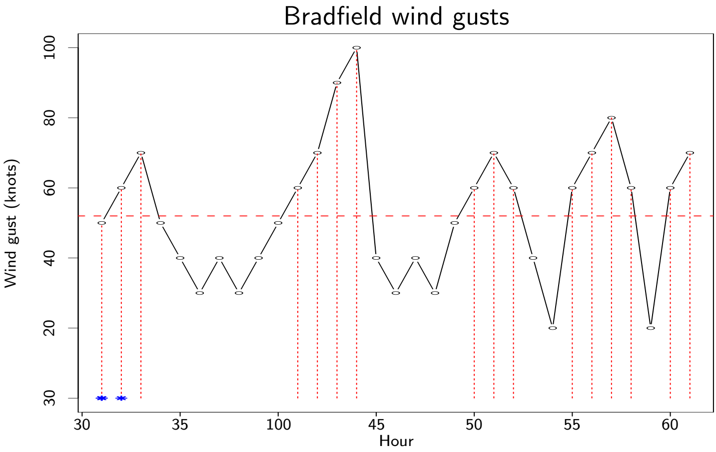

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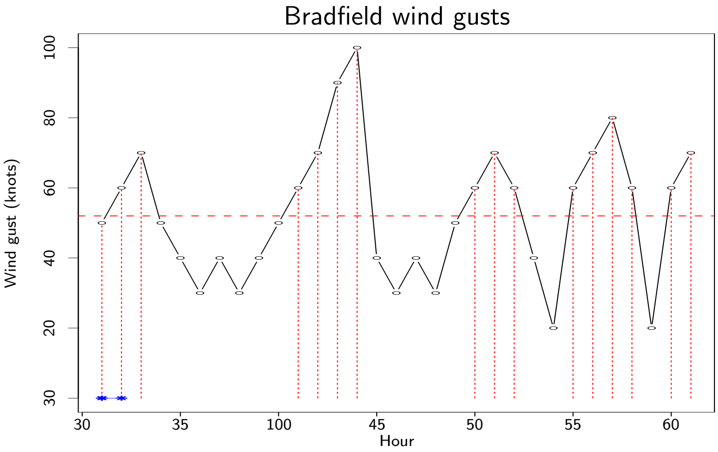

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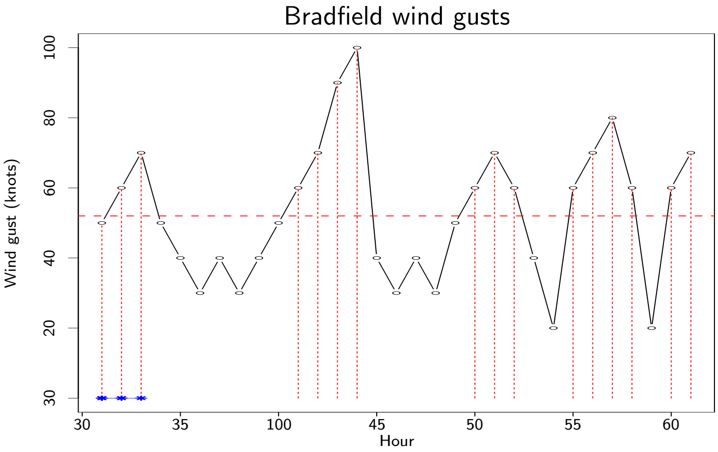

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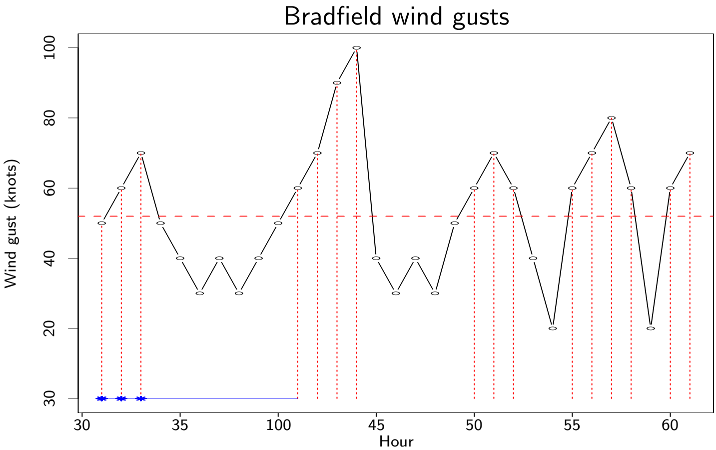

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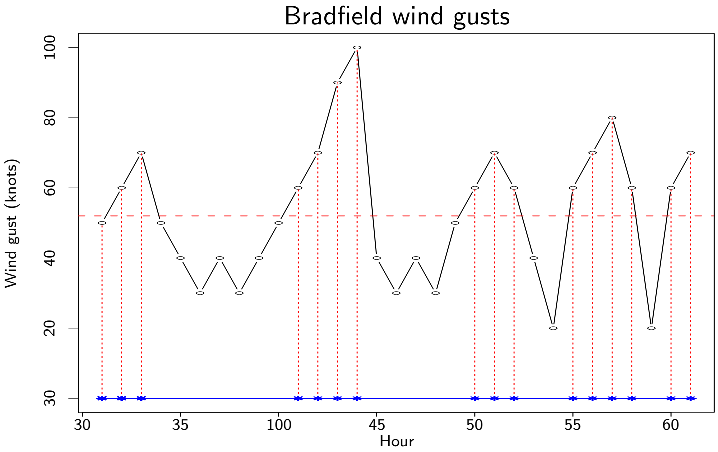

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We now combine the likelihood from the maxima method (Ancona–Navarrete and Tawn, 2000), with the 'automatic' declustering procedure (Ferro and Segers, 2003), to implement a Bayesian sampling scheme for θ and any other cluster functional of interest:

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5. use each set of identified clusters found using $\kappa^{(r)}$, $r = 1, \ldots, R$, to estimate any other cluster characteristic, say $H^{(r)}$, and so obtain draws from the (approximate) posterior distribution for that functional also.

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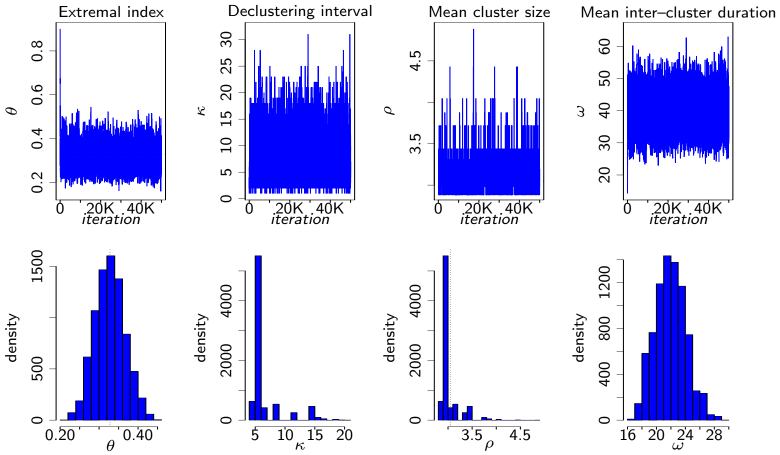
We use each posterior draw for θ to obtain a corresponding draw for κ; each κ implements a full declustering procedure from which we can observe the (posterior) distribution for *any* cluster characteristic!

Simulated data



Simulated data

Some numerical summaries (after burn–in)

	θ ($= 0.062$)	κ	ρ	ω
Posterior mean (s.d.)	0.065 (0.050)	14.035 (4.002)	14.648 (0.707)	53.343 (10.546)
95% credible interval	(0.031, 0.165)	(6, 23)	(12.203, 18.321)	(32.222, 77.372)
m.l.e. (asymp. s.e.)	0.058 (0.052)	—	13.317 (1.193)	—

	θ ($= 0.328$)	κ	ρ	ω
Posterior mean (s.d.)	0.319 (0.048)	5.974 (3.678)	3.048 (0.237)	39.662 (5.440)
95% credible interval	(0.225, 0.416)	(2, 16)	(2.618, 3.420)	(29.715, 50.867)
m.l.e. (asymp. s.e.)	0.297 (0.047)	—	3.361 (0.233)	—

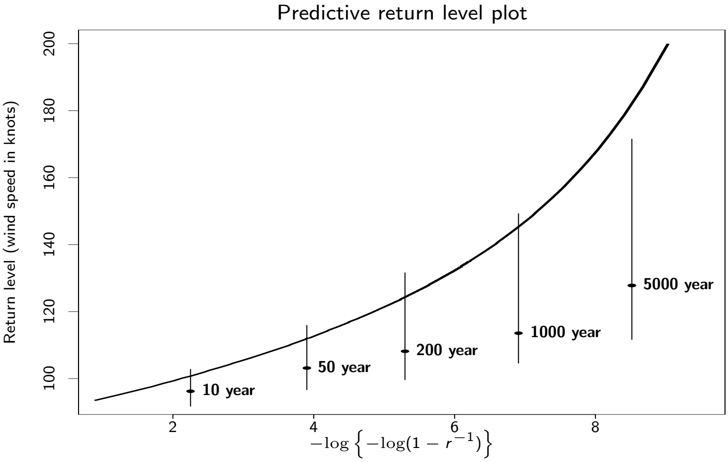
Bradfield wind speed data

Some numerical summaries for January (after burn–in)

	θ	κ	Mean storm length	Mean time between storms
Posterior mean (s.d.)	0.243 (0.047)	5.266 (5.840)	4.924 (0.637)	82.747 (15.271)
95% credible interval	(0.162, 0.347)	(2, 24)	(4.289, 6.246)	(56.435, 117.259)
m.l.e. (asymp. s.e.)	0.207 (0.042)	—	4.833 (0.578)	—

Bradfield wind speed data

Return level inference



References

Ancona–Navarrete, M.A. and Tawn, J.A. (2000). A Comparison of Methods for Estimating the Extremal Index. *Extremes*, **3**, pp. 5—38.

Fawcett, L. and Walshaw, D. (2008). Bayesian Inference for Clustered Extremes. *Extremes*, **11**, pp. 217—233.

Fawcett, L. and Walshaw, D. (2006). Markov chain models for extreme wind speeds. *Environmetrics*, **17**, pp. 795—809.

Ferro, C.A.T. and Segers, J. (2003). Inference for clusters of extreme values. *J. R. Statist. Soc., B*, **65**, pp. 545—556.

Hsing, T., Hüsler, J. and Leadbetter, M.R. (1988). On the exceedance point process for a stationary sequence. *Prob. Theory Rel. Fields* **78**, pp. 97—112.

Leadbetter, M.R., Lindgren, G. and Rootzén, H. (1983). *Extremes and Related Properties of Random Sequences and Series*. Springer–Verlag, New York.

Smith, R.L. (1992). The Extremal Index for a Markov Chain. *J. Appl. Prob.*, **29**, pp. 37—45.

Smith, R.L., Coles, S.G. and Tawn, J.A. (1997). Markov chain models for threshold exceedances. *Biometrika*, **84**,

pp. 249—268.