Minitab Practicals

- This week tutorials are replaced with `Minitab` practicals
- Same time (except group F) – just in a computer cluster instead
  - WED 9 – Herschel
  - WED 10 – Herschel
  - WED 11 – Herschel
  - THUR 9 – Medical School
  - THUR 12 – Medical School
  - THUR 3 – THUR 1 – Medical School
  - FRI 9 – Medical School
  - FRI 11 – Medical School
- Make sure you attend your alloacted slot!

Optional (extra) workshop on hypothesis testing this week, **Wednesday, 2pm, Herschel LT3**.

- Worksheets given out (not available again!)
- Summary sheets given out which you can take into the exam
- Topics I intend to cover:
  - Hypothesis tests for one mean
  - Hypothesis tests for two means
  - When is it case 1 or case 2?
  - When do I use a one/two–tailed test?
  - Chi–squared goodness–of–fit tests
  - Chi–squared tests for independence

Lecture 7

USING MINITAB

Hypothesis tests for one mean

Recall the hypothesis tests for one mean (section 2.3).

Here, from a single population, we draw a single sample, and we estimate the population mean μ with $\bar{x}$.

We'd then like to see how convincing a proposal for the population mean is, based on the information in our sample.

Our null hypothesis is

$$H_0 \quad : \quad \mu = c.$$

The alternative could be

$$\begin{aligned} H_1 &: \mu \neq c, \\ H_1 &: \mu > c \qquad \text{or} \\ H_1 &: \mu < c. \end{aligned}$$

Hypothesis tests for one mean

The test statistic we calculate, and the statistical tables we consult to obtain our p–value, depend on whether or not the population variance is known. If the population variance (σ^2) is *known*, we use the test statistic

$$z = \frac{|\bar{x} - \mu|}{\sqrt{\sigma^2/n}}.$$

If the population variance is unknown, we use

$$t = \frac{|\bar{x} - \mu|}{\sqrt{s^2/n}},$$

where s^2 is the *sample* variance.

Hypothesis tests for one mean

If the population variance is known, the test statistic is called z; another name for this hypothesis test is therefore the **one–sample z test**.

If the population variance is unknown, the test statistic is called t, and we consult tables of probabilities for the t–distribution to obtain our critical value; another name for this hypothesis test is the **one–sample t test**.

Hypothesis tests for one mean

This example is the same as that in **question 3** of the **exercises in chapter 2**.

A company is in dispute with its workforce. The workers claim that under a new flexitime system they are working longer than the standard 37.5 hours per week. The time cards of 10 workers were selected at random and these showed the following hours worked:

35 40 45 41 36 37 39 38 42 32

Are the staff working more than a standard week?

The hypotheses are:

$$H_0 : \mu = 37.5 \quad \text{versus}$$
$$H_1 : \mu > 37.5$$

Let's see this in action in `Minitab`!

Hypothesis tests for one mean

So what do **we** need to do? And what does `Minitab` do?

- We still need to set up the hypotheses
- We don't need to calculate the test statistic – `Minitab` does that for us
- We don't need to use tables to find the *p*–value – `Minitab` does that as well
- In fact, instead of us saying "the *p*–value lies between 5% and 10%" or "the *p*–value is bigger than 10%", `Minitab` gives us a **precise *p*–value**
- `Minitab` gives the *p*–value as a decimal – you'll probably want to convert this to a percentage ($\times 100$)
- We still need to form our conclusions – `Minitab` does not do that!

Hypothesis tests for one mean

When we did this example by hand, we got:

- Test statistic: $t = 0.84$
- p–value bigger than 10%

Using `Minitab`, we get:

- Test statistic: $t = 0.84$
- $p = 0.210 = 21\%$

Therefore:

- We have **no** evidence against H_0
- We **retain** H_0
- It appears that, on average, the workers have nothing to complain about!

Tests for two means

If we have **two** independent random samples from **two** populations, we can compare the two sample means.

The null hypothesis for such a test is

$$H_0 \quad : \quad \mu_1 = \mu_2.$$

The alternative hypothesis could be one of

$$\begin{aligned} H_1 &: \mu_1 \neq \mu_2, \\ H_1 &: \mu_1 > \mu_2 \qquad \text{or} \\ H_1 &: \mu_1 < \mu_2. \end{aligned}$$

Tests for two means

The formula we use for the test statistic will depend on whether or not both population variances are known. In the case where both *are* known, we use

$$z = \frac{|\bar{x}_1 - \bar{x}_2|}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}.$$

If both are unknown, we use

$$t = \frac{|\bar{x}_1 - \bar{x}_2|}{s \times \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}},$$

where s is the "pooled standard deviation" (see section 3.2.2). Thus, similar to the one–sample case, we have a **two–sample z test** if both population variances are known, and a **two–sample t test** if both are unknown.

Tests for two means

For example, if we compare the average flight times (in minutes) of two flight companies between Newcastle and Palma, we might observe the following data:

EasyJet	150	147	141	158	155	133	140
Britannia	162	158	163	152	156	149	150

To test for a general difference in the flight times between the two companies, we test the null hypothesis

$$\begin{aligned} H_0 &: \mu_1 = \mu_2 \qquad \text{against} \\ H_1 &: \mu_1 \neq \mu_2. \end{aligned}$$

Tests for two means

Since the p–value is 0.035, or 3.5%, it lies between 1% and 5% and so we have **moderate** evidence to **reject** the null hypothesis in favour of the alternative.

It appears the two flight companies *do* have different flight times on this route (and Britannia's look longer).

We can also perform test of independence in `Minitab` (see chapter 5 of these notes).

Going back to the employment status and gender example, we have the following contingency table of observed frequencies:

	Permanent	Temporary	Unemployed	Total
Male	100	33	25	158
Female	90	40	22	152
Total	190	73	47	310

Tests of independence using the χ^2 distribution

The aim here was to test for an association between the two categorical variables. The null and alternative hypotheses are

H_0 : There is no association between employment status and gender

H_1 : There *is* an association between employment status and gender.

Tests of independence using the χ^2 distribution

From the `Minitab` output, we see that our p–value is 0.529, or 52.9%.

- There is **no** evidence against H_0
- We should therefore **retain** H_0
- It appears that there is no association between employment status and gender (again, exactly the same as when we performed this test by hand; compare `Minitab`'s calculations to those on pages 45–47).

Last week we looked at how to calculate **correlation coefficients** and perform a **regression analysis** on paired data.

Well, guess what – `Minitab` can do that as well!

Consider the ice cream sales data from last week. If we enter the average temperatures in column 1 of a new `Minitab` worksheet, and ice cream sales in the other, we can use `Minitab`'s correlation and regression options to do everything we did by hand in last week's lecture.

When using `Minitab` to calculate the correlation coefficient, this is the output we get:

Correlations: C1,C2

```
Pearson correlation of C1 and C2 = 0.983
P-Value = 0.000
```

Notice that we also get a p–value for the correlation coefficient. In fact, this is associated with the following null hypothesis:

$$H_0 \quad : \quad \text{Population correlation coefficient} = 0$$