Lecture 3

HYPOTHESIS TESTS FOR TWO MEANS

Announcements

CBA4 goes live in practice mode this week – exam mode next week

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Assignment 1 feedback

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Assignment 1 feedback

Mean	St. dev.	Median	IQR	95% CI	Missing
89.9	10.2	94.0	11.1	(89.1, 90.9)	27

Introduction

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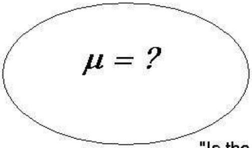
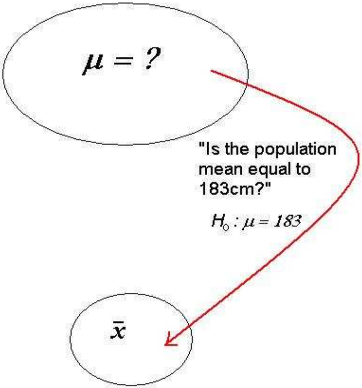
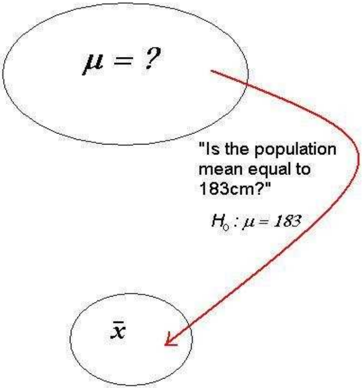
If we have **two** independent random samples from **two** populations, we can compare the two sample means in a test for two means (*c.f.* comparing one sample mean to a *proposed value* in the one–sample case).



"Is the population mean equal to 183cm?"

$$H_0 : \mu = 183$$


$\mu = ?$
"Is the population mean equal to 183cm?"
$H_0 : \mu = 183$
$\bar{x}$



$\mu = ?$
"Is the population mean equal to 183cm?"
$H_0 : \mu = 183$
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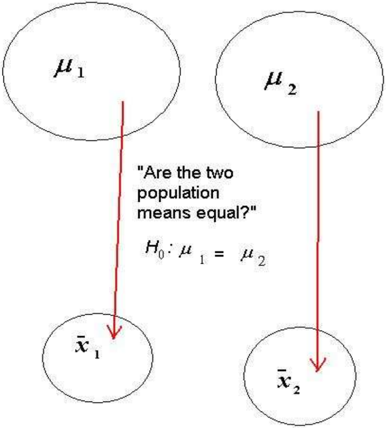

μ_1
μ_2
"Are the two population means equal?"
$H_0 : \mu_1 = \mu_2$
$\bar{x}_1$
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However, the calculations required for the test statistic in step 3 are slightly different.

Testing two means

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- **case 1:** both population variances **known** and
- **case 2:** both population variances **unknown**.

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   We usually test against the (two–tailed) alternative:

$$H_1 \quad : \quad \mu_1 \neq \mu_2,$$

   i.e. the population means *are not* equal. However, we might use the one–tailed alternatives:

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   $$\begin{aligned} H_1 &: \quad \mu_1 > \mu_2, \qquad \text{or} \\ H_1 &: \quad \mu_1 < \mu_2. \end{aligned}$$

3. Calculate the test statistic

The test statistic for a two–sample test (when both population variances are known) is

$$z = \frac{|\bar{x}_1 - \bar{x}_2|}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

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4. **Find the p–value**

   This is found from statistical tables; since, in this case, both population variances σ_1^2 and σ_2^2 are *known*, we refer to standard normal tables.

   As before, we find a range for our p–value by comparing our test statistic to the 10%, 5% and 1% critical values.

5. **Form a conclusion**
   Exactly the same again! Use table 2.1 to help you decide what to do! Word your conclusions in the context of the original question.

Example (page 25)

Before a training session for call centre employees a sample of 50 calls to the call centre had an average duration of 5 minutes, whereas after the training session a sample of 45 calls had an average duration of 4.5 minutes.

The population variance is known to have been 1.5 minutes before the course and 2 minutes afterwards.

Has the course been effective?

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Example (page 25)

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$$z = \frac{|\bar{x}_B - \bar{x}_A|}{\sqrt{\frac{\sigma_B^2}{n_B} + \frac{\sigma_A^2}{n_A}}} = \frac{|5 - 4.5|}{\sqrt{\frac{1.5}{50} + \frac{2}{45}}}$$

Example (page 25)

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$$\begin{aligned} z &= \frac{|\bar{x}_B - \bar{x}_A|}{\sqrt{\frac{\sigma_B^2}{n_B} + \frac{\sigma_A^2}{n_A}}} \\ &= \frac{|5 - 4.5|}{\sqrt{\frac{1.5}{50} + \frac{2}{45}}} \\ &= 1.833 \end{aligned}$$

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Example (page 25)

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Since $z = 1.833$ lies between 1.645 and 1.96, our p–value lies between 5% and 10%.

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Example (page 25)

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Example (page 25)

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Using table 2.1 (last week), this means that:

- we have **slight** evidence against H_0
- This is not small enough to reject H_0, and so we **retain** H_0
- There is insufficient evidence to suggest that the training has had any affect on the average duration of a call.

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Like before, we have to use t–tables to obtain our critical value; the degrees of freedom is now found as $\nu = n_1 + n_2 - 2$.

Example (page 26)

A company is interested in knowing if two branches have the same level of average transactions. The company sample a small number of transactions and calculates the following statistics:

Shop 1	$\bar{x}_1 = 130$	$s_1^2 = 700$	$n_1 = 12$
Shop 2	$\bar{x}_2 = 120$	$s_2^2 = 800$	$n_2 = 15$

Test whether or not the two branches have (on average) the same level of transactions.

Example (page 26)

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Our null and alternative hypotheses are:

$$\begin{aligned} H_0 &: \mu_1 = \mu_2 \qquad \text{versus} \\ H_1 &: \mu_1 \neq \mu_2. \end{aligned}$$

Example (page 26)

Step 3 *(calculating the test statistic)*
Since both population variances are unknown (only the *sample* values are given), the test statistic is

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Example (page 26)

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Step 4 *(finding the p–value)*
Since both population variances are unknown, we use t–tables to obtain our critical value.

The degrees of freedom is

$$\begin{aligned} \nu &= n_1 + n_2 - 2 \qquad \text{i.e} \\ &= 12 + 15 - 2 \\ & \ 25. \end{aligned}$$

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Critical value	1.708	2.060	2.787

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Critical value	1.708	2.060	2.787

Our test statistic $t = 0.939$ lies to the left of the first critical value, and so our p–value is bigger than 10%.

Example (page 26)

Step 5 *(conclusion)*
Using table 2.1, we see that, since our p–value is larger than 10%, we have no evidence to reject the null hypothesis. Thus, we retain H_0 and conclude that there is no significant difference between the average level of transactions at the two shops.