

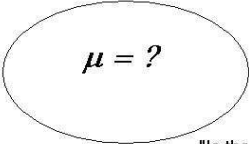
Lecture 2

HYPOTHESIS TESTS FOR TWO MEANS

Introduction

Last week you were introduced to the concept of hypothesis testing in statistics, and we considered hypothesis tests for the mean if we have a **single** sample drawn from a **single** population.

If we have **two** independent random samples from **two** populations, we can compare the two sample means in a test for two means (*c.f.* comparing one sample mean to a *proposed value* in the one-sample case).


$$\mu = ?$$

"Is the population
mean equal to
183cm?"

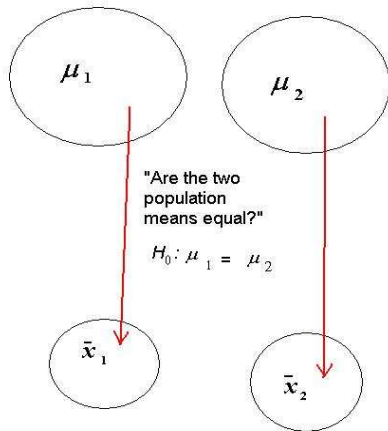
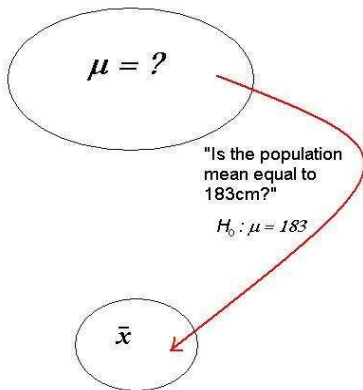
$$H_0 : \mu = 183$$

$$\mu = ?$$

"Is the population
mean equal to
183cm?"

$$H_0 : \mu = 183$$

$$\bar{x}$$



We use the same framework for hypothesis testing as for the one-sample tests:

1. State the **null hypothesis**, H_0 ;
2. State the **alternative hypothesis**, H_1 ;
3. Calculate a **test statistic**;
4. Find the **p-value**, and
5. Use table 2.1 to form your **conclusions**.

However, the calculations required for the test statistic in step 3 are slightly different.

Testing two means

Recall that, in the test for one mean, there were two cases:
population variance (σ^2) known and population variance unknown.

Similarly, when comparing two means, we can consider

- **case 1:** both population variances **known** and
- **case 2:** both population variances **unknown**.

Both population variances (σ_1^2 and σ_2^2) known

1. State the null hypothesis

This time, the null hypothesis is

$$H_0 : \mu_1 = \mu_2,$$

i.e. the two population means are equal.

2. State the alternative hypothesis

We usually test against the (two-tailed) alternative:

$$H_1 : \mu_1 \neq \mu_2,$$

i.e. the population means *are not* equal. However, we might use the one-tailed alternatives:

$$H_1 : \mu_1 > \mu_2, \quad \text{or}$$

$$H_1 : \mu_1 < \mu_2.$$

3. Calculate the test statistic

The test statistic for a two-sample test (when both population variances are known) is

$$z = \frac{|\bar{x}_1 - \bar{x}_2|}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

4. Find the p-value

This is found from statistical tables; since, in this case, both population variances σ_1^2 and σ_2^2 are *known*, we refer to standard normal tables.

As before, we find a range for our p -value by comparing our test statistic to the 10%, 5% and 1% critical values.

5. Form a conclusion

Exactly the same again! Use table 2.1 to help you decide what to do! Word your conclusions in the context of the original question.

Example (page 25)

Before a training session for call centre employees a sample of 50 calls to the call centre had an average duration of 5 minutes, whereas after the training session a sample of 45 calls had an average duration of 4.5 minutes.

The population variance is known to have been 1.5 minutes before the course and 2 minutes afterwards.

Has the course been effective?

Steps 1 and 2 (*hypotheses*)

We test

$$H_0 : \mu_B = \mu_A \quad \text{against}$$

$$H_1 : \mu_B \neq \mu_A$$

Step 3 (*test statistic*)

Since both population variances are known, we use

$$\begin{aligned} z &= \frac{|\bar{x}_B - \bar{x}_A|}{\sqrt{\frac{\sigma_B^2}{n_B} + \frac{\sigma_A^2}{n_A}}} \\ &= \frac{|5 - 4.5|}{\sqrt{\frac{1.5}{50} + \frac{2}{45}}} \\ &= 1.833 \end{aligned}$$

Step 4 (*p*-value)

We used a **two-tailed** alternative; using table 2.2, we get:

Significance level	10%	5%	1%
Critical value	1.645	1.96	2.576

Since $z = 1.833$ lies between 1.645 and 1.96, our *p*-value lies between 5% and 10%.

Step 5 (*conclusion*)

Using table 2.1 (last week), this means that:

- we have **slight** evidence against H_0
- This is not small enough to reject H_0 , and so we **retain** H_0
- There is insufficient evidence to suggest that the training has had any affect on the average duration of a call.

Both population variances (σ_1^2 and σ_2^2) unknown

In the more likely situation where the population variances are unknown, the test statistic becomes

$$t = \frac{|\bar{x}_1 - \bar{x}_2|}{s \times \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}},$$

where s is a “pooled standard deviation”, and is found as

$$s = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}.$$

Like before, we have to use t -tables to obtain our critical value; the degrees of freedom is now found as $\nu = n_1 + n_2 - 2$.

Example (page 26)

A company is interested in knowing if two branches have the same level of average transactions. The company sample a small number of transactions and calculates the following statistics:

$$\begin{array}{l|l} \text{Shop 1} & \bar{x}_1 = 130 \quad s_1^2 = 700 \quad n_1 = 12 \\ \text{Shop 2} & \bar{x}_2 = 120 \quad s_2^2 = 800 \quad n_2 = 15 \end{array}$$

Test whether or not the two branches have (on average) the same level of transactions.

Steps 1 and 2 (*hypotheses*)

Our null and alternative hypotheses are:

$$H_0 : \mu_1 = \mu_2 \quad \text{versus}$$

$$H_1 : \mu_1 \neq \mu_2.$$

Example (page 26)

Step 3 (*calculating the test statistic*)

Since both population variances are unknown (only the *sample* values are given), the test statistic is

$$t = \frac{|\bar{x}_1 - \bar{x}_2|}{s \times \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}};$$

thus, we first need to obtain the pooled variance s . This is given as

$$\begin{aligned} s &= \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}} \\ &= \sqrt{\frac{11 \times 700 + 14 \times 800}{25}} \\ &= \sqrt{\frac{7700 + 11200}{25}} \\ &= 27.495. \end{aligned}$$

Thus,

$$\begin{aligned}t &= \frac{|130 - 120|}{27.495 \times \sqrt{\frac{1}{12} + \frac{1}{15}}} \\&= \frac{10}{27.495 \times \sqrt{0.15}} \\&= \frac{10}{10.649} \\&= 0.939.\end{aligned}$$

Example (page 26)

Step 4 (*finding the p -value*)

Since both population variances are unknown, we use t -tables to obtain our critical value.

The degrees of freedom is

$$\begin{aligned}\nu &= n_1 + n_2 - 2 \quad \text{i.e.} \\ &= 12 + 15 - 2 \\ &25.\end{aligned}$$

Under a two-tailed test, and using table 2.3, we get the following critical values:

Significance level	10%	5%	1%
Critical value	1.708	2.060	2.787

Our test statistic $t = 0.939$ lies to the left of the first critical value, and so our p -value is bigger than 10%.

Step 5 (*conclusion*)

Using table 2.1, we see that, since our p -value is larger than 10%, we have no evidence to reject the null hypothesis. Thus, we retain H_0 and conclude that there is no significant difference between the average level of transactions at the two shops.