NEWCASTLE UNIVERSITY

SCHOOL OF MATHEMATICS & STATISTICS

SEMESTER 2 2007/2008

MAS1403

Quantitative Methods for Business Management

Time allowed: 2 hours

Credit will be given for ALL answers to questions in Section A, and for the best TWO answers to questions in Section B. No credit will be given for other answers and students are strongly advised not to spend time producing answers for which they will receive no credit.

Marks for each question are indicated. However you are advised that marks indicate the relative weight of individual questions, they do not correspond directly to marks on the University scale.

There are FOUR questions in Section A and FOUR questions in Section B.

Answers to questions in Section A should be entered directly on this question paper in the spaces provided. Rough work should be done on the blank sides of the pages. The rough work will not be marked. This question paper must be handed in, attached inside an anonymised cover sheet, at the end of the examination.

Calculators may be used. Graph paper will be provided. This examination is "open–book"; students may consult their notes, text books etc during the examination.

SECTION A

A1. The following data are total monthly sales (April 2008, in thousands of pounds) for an electrical store with branches in ten cities in England:

32 43 38 53 43 15 27 52 25 42

(a) Calculate the mean, median and standard deviation for these sales.

Answer:

(b) In the space below construct a box–and–whisker plot for the sales data. *[Hint: The lower and upper quartiles are 26.5 sales and 45.25 sales, respectively]*

Answer:

[9 marks]

A2. The *Holiday Hypermarket* call centre can receive, at most, fifteen calls in any one minute period (otherwise customers hear an "engaged" tone). Experience has shown that 30% of all customers who do not hear the engaged tone are put 'on hold' until an operator can take their call.

(a) What probability distribution might be reasonable to use to model the number of customers who are put 'on hold' in any one minute period?

Answer:

(b) What is the expected number and standard deviation of the number of customers put 'on hold'?

Answer:

(c) What is the probability that during any one minute period, exactly five customers are put 'on hold'?

Answer:

(d) The time between successful telephone sales at the *Holiday Hypermarket* can be assumed to follow an exponential distribution with rate $\lambda = 2$ per minute. Find the probability that the time between two consecutive sales is less than fifteen seconds.

Answer:

[12 marks]

A3. *Green Valley Foods* produce bags of frozen vegetables with a mean weight of 1kg. The company decides that all bags weighing more than 1060g will be repacked. Assuming the weight of the bags is Normally distributed with standard deviation 25g, what percentage of bags will be repacked?

Answer:

[**7 marks**]

A4. The following data gives the total market value of 8 companies (X, in millions of pounds) and the number of stock exchange transactions (Y) in that company's shares occurring on a particular day.

x	6.3	5.5	0.3	1.5	1.7	2.8	3.6	3.9
y	330	180	10	60	40	50	70	120

Use the following summaries to find the sample correlation coefficient.

$$\sum x = 25.6 \qquad \sum y = 860$$

$$\sum x^2 = 111.18 \qquad \sum y^2 = 168400 \qquad \sum xy = 4090$$

Answer:

[**9 marks**]

A5. (a) The manager of a hotel claims that the average guest bill for a weekend is £350. A local journalist, however, believes that prices have increased and that average bills are greater than £350.

To investigate, the journalist took a sample of 25 bills from customers who had recently stayed at the hotel; the mean from this sample was $\bar{x}$ = £385. The next evening, an article appeared in the local newspaper with the following headline:

LOCAL HOTEL MORE EXPENSIVE THAN IT CLAIMS!

The sample standard deviation was s = £98.50. Calculate a 95% confidence interval for the mean guest bill, and comment on the validity of the newspaper headline.

Answer:

Question A5 continues on the next page

(b) A sample of 20 weekend–stay guest bills from another hotel gave a sample mean of $\bar{x} = £435$ with a standard deviation of $s = £72.50$. Perform an appropriate hypothesis test to determine whether there is a difference in the mean guest bill between the two hotels. *[Hint: the pooled standard deviation is £87.96]*

Answer:

[**13 marks**]

SECTION B

B6. The regional manager of a chain of 100 estate agents is considering implementing a new staff bonus scheme. This 'bonus' scheme is, in fact, linked to the total number of customer complaints each branch receives (X per month), and if more than 2 complaints are received, a negative 'bonus' is applied to the overall staff salary budget for that branch. The 'bonus' each branch receives is shown below, along with the observed number of branches receiving complaints in April 2008.

No. of complaints (X)	Bonus (£)	Frequency (April 2008)
0	1000	12
1	750	61
2	200	20
3	–500	7
4+	–1500	0

(a) Using the frequencies in the table above, calculate the mean number of complaints received in April 2008.

(b) Propose an appropriate probability model for the number of complaints received.

(c) Using the mean you calculated in part (a), and the model identified in part (b), obtain the full probability distribution for the number of complaints, i.e. $P(X = 0)$, $P(X = 1)$, $P(X = 2)$, $P(X = 3)$ and $P(X \geq 4)$.

(d) Using the probabilities you found in part (c), and the bonuses shown in the table above, calculate the regional manager's *Expected Monetary Value* for this bonus scheme.

(e) Perform a goodness–of–fit test to see if the data observed are consistent with the model you proposed in part (b). *[Hint: Use the probabilities in part (c) to obtain your expected frequencies for this test]* Comment on the validity of your *EMV* calculation in part (d).

[25 marks]

B7. *Pizza Classic* are an Italian catering company specialising in pizza and pasta orders for functions and events. Each of their "Pizza Classica" pizzas uses 3 tins of tomatoes and 8 mozzarella balls. Each of their "Pasta Classica" dishes uses 9 tins of tomatoes and 4 mozzarella balls. Each week, the company has 4500 tins of tomatoes and 4000 mozzarella balls available. Current demand requires that the company makes at least 100 of their pasta dishes, and twice as many pizzas, each week. The company sells each "Pizza Classica" pizza at a profit of £4 and each "Pasta Classica" dish at a profit of £5. The company makes and sells x "Pizza Classica" pizzas and y "Pasta Classica" dishes each week.

(a) Formulate *Pizza Classic*'s situation as a linear programming problem.

(b) Draw a suitable diagram to enable the problem to be solved graphically, indicating the feasible region and the direction of the objective line.

(c) Using your diagram, find the company's minimum and maximum weekly profit, £P.

(d) Now solve this problem algebraically to verify the company's maximum weekly profit obtained in part (c).

[25 marks]

B8. *Bowood House* is a British Stately Home in Wiltshire, built in the 1750s. It is open to the public for six hours a day (10am–4pm). The table below shows Y, the total number of visitors, over the last three years for the "morning" period (10am–12 noon), "lunchtime" period (12noon–2pm) and the "afternoon" period (2pm–4pm).

	Morning	Lunchtime	Afternoon
2005	2350	2620	3100
2006	1700	1940	2520
2007	1020	1350	1800

(a) Produce a time series plot for these data, and comment.

(b) Calculate the moving averages for these data, and overlay these on your plot in part (a).

(c) Let T represent time, and let time–points t_i, $i = 1, 2, \ldots$, correspond to Morning (2005), Lunchtime (2005), ... (respectively). Also, let y_i be the moving average at time t_i, as calculated in part (b). Then we have the following summaries:

$$\sum_{i=2}^{8} t_i = 35 \qquad \sum_{i=2}^{8} y_i = 14310$$

$$\sum_{i=2}^{8} t_i^2 = 203 \qquad \sum_{i=2}^{8} t_i y_i = 65543.33$$

Use the summaries above to estimate the linear trend

$$Y = \alpha + \beta T + \epsilon.$$

(d) The (unadjusted) seasonal effects are given in the table below.

"Season"	Seasonal effects
Morning	–569.7
Lunchtime	–65.9
Afternoon	+668.6

Obtain the *adjusted* seasonal effects.

(e) The family who own *Bowood House* have decided to close the doors to the public between 10am and 12 noon if the predicted annual number of visitors falls below 500. Use the linear trend equation in part (c), and the adjusted seasonal effects in part (d), to forecast the total number of visitors to *Bowood House* between 10am and 12 noon this year (2008). Advise the family accordingly.

[25 marks]

B9. You are a Business Analyst working for a travel company that specialises in "Gap Year" adventure holidays for students and those taking career breaks. The table below shows the amount (X hundred pounds) customers with this travel company paid for their holidays in 2007.

	Frequency		**Percentage**	
X	Students	Career–breakers	Students	Career–breakers
$10 \le x < 14$	80	20	7.0	
$14 \le x < 18$	120	30	10.5	
$18 \le x < 22$	250	35	21.8	
$22 \le x < 26$	330	60	28.8	
$26 \le x < 30$	220	90	19.2	
$30 \le x < 34$	95	210	8.3	
$34 \le x < 38$	50	100	4.4	
$38 \le x < 42$	0	70	0	
Totals	1145	615	100	

(a) Complete the table by calculating the percentage values for the "Career–Breakers" group.

(b) You decide to use polygons to compare customer expenditure between the two groups. Should you use *frequency* or *relative (percentage) frequency* polygons? Explain.

(c) On graph paper, produce the most appropriate graph (as identified in part (b) above) to compare customer expenditure between the two groups. Comment.

(d) Use the frequency table above to estimate the mean expenditure for students. Why is your answer an *approximation* of the sample mean?

(e) The standard deviation for student expenditure can be estimated from the above table as £588.37. Assuming that student expenditure follows a Normal distribution with a mean as calculated in part (d) and a standard deviation of £588.37, find the probability that a randomly chosen student will spend at least £3500 on her adventure holiday. Comment on the validity of this calculation.

(f) Would you trust such a probability calculation for the "Career–breakers" group? Explain.

[25 marks]

THE END

Statistical tables overleaf →

Statistical tables

		p				
	One–tailed test	10%	5%	2.5%	1%	0.5%
	Two–tailed test	20%	10%	5%	2%	1%
	1	3.078	6.314	12.706	31.821	63.657
	2	1.886	2.920	4.303	6.965	9.925
	3	1.638	2.353	3.182	4.541	5.841
	4	1.533	2.132	2.776	3.747	4.604
	5	1.476	2.015	2.571	3.365	4.032
	6	1.440	1.943	2.447	3.143	3.707
	7	1.415	1.895	2.365	2.998	3.449
	8	1.397	1.860	2.306	2.896	3.355
	9	1.383	1.833	2.262	2.821	3.250
	10	1.372	1.812	2.228	2.764	3.169
	11	1.363	1.796	2.201	2.718	3.106
	12	1.356	1.782	2.179	2.681	3.055
	13	1.350	1.771	2.160	2.650	3.012
	14	1.345	1.761	2.145	2.624	2.977
ν	15	1.341	1.753	2.131	2.602	2.947
	16	1.337	1.746	2.120	2.583	2.921
	17	1.333	1.740	2.110	2.567	2.898
	18	1.330	1.734	2.101	2.552	2.878
	19	1.328	1.729	2.093	2.539	2.861
	20	1.325	1.725	2.086	2.528	2.845
	21	1.323	1.721	2.080	2.518	2.831
	22	1.321	1.717	2.074	2.508	2.819
	23	1.319	1.714	2.069	2.500	2.807
	24	1.318	1.711	2.064	2.492	2.797
	25	1.316	1.708	2.060	2.485	2.787
	26	1.315	1.706	2.056	2.479	2.779
	27	1.314	1.703	2.052	2.473	2.771
	28	1.313	1.701	2.048	2.467	2.763
	29	1.311	1.699	2.045	2.462	2.756
	⋮	⋮	⋮	⋮	⋮	⋮
	∞	1.282	1.645	1.960	2.326	2.576

Table 1: This table contains values of t for which $\Pr(T > t) = p$, where $T \sim t_\nu$. In a two–tailed test, the tabulated values correspond to $\Pr(|T| > t) = p$

		p				
		50%	10%	5%	1%	0.1%
	1	0.45	2.17	3.84	6.63	10.83
	2	1.39	4.61	5.99	9.21	13.82
	3	2.37	6.25	7.82	11.34	16.27
	4	3.36	7.78	9.49	13.28	18.47
	5	4.34	9.24	11.07	15.09	20.52
	6	5.35	10.64	12.59	16.81	22.46
	7	6.35	12.02	14.07	18.48	24.32
ν	8	7.34	13.36	15.51	20.09	26.13
	9	8.34	14.68	16.92	21.67	27.88
	10	9.34	15.99	18.31	23.21	29.59
	12	11.34	18.55	21.03	26.22	32.91
	15	14.34	22.31	25.00	30.58	37.70
	20	19.34	28.41	31.41	37.57	45.32
	25	24.34	34.38	37.65	44.31	52.62
	30	29.34	40.26	43.77	50.89	59.70

Table 2: This table contains values of x for which $\Pr(X^2 > x) = p$, where $X^2 \sim \chi^2_\nu$

z	-0.09	-0.08	-0.07	-0.06	-0.05	-0.04	-0.03	-0.02	-0.01	0.00
-2.9	0.0014	0.0014	0.0015	0.0015	0.0016	0.0016	0.0017	0.0018	0.0018	0.0019
-2.8	0.0019	0.0020	0.0021	0.0021	0.0022	0.0023	0.0023	0.0024	0.0025	0.0026
-2.7	0.0026	0.0027	0.0028	0.0029	0.0030	0.0031	0.0032	0.0033	0.0034	0.0035
-2.6	0.0036	0.0037	0.0038	0.0039	0.0040	0.0041	0.0043	0.0044	0.0045	0.0047
-2.5	0.0048	0.0049	0.0051	0.0052	0.0054	0.0055	0.0057	0.0059	0.0060	0.0062
-2.4	0.0064	0.0066	0.0068	0.0069	0.0071	0.0073	0.0075	0.0078	0.0080	0.0082
-2.3	0.0084	0.0087	0.0089	0.0091	0.0094	0.0096	0.0099	0.0102	0.0104	0.0107
-2.2	0.0110	0.0113	0.0116	0.0119	0.0122	0.0125	0.0129	0.0132	0.0136	0.0139
-2.1	0.0143	0.0146	0.0150	0.0154	0.0158	0.0162	0.0166	0.0170	0.0174	0.0179
-2.0	0.0183	0.0188	0.0192	0.0197	0.0202	0.0207	0.0212	0.0217	0.0222	0.0228
-1.9	0.0233	0.0239	0.0244	0.0250	0.0256	0.0262	0.0268	0.0274	0.0281	0.0287
-1.8	0.0294	0.0301	0.0307	0.0314	0.0322	0.0329	0.0336	0.0344	0.0351	0.0359
-1.7	0.0367	0.0375	0.0384	0.0392	0.0401	0.0409	0.0418	0.0427	0.0436	0.0446
-1.6	0.0455	0.0465	0.0475	0.0485	0.0495	0.0505	0.0516	0.0526	0.0537	0.0548
-1.5	0.0559	0.0571	0.0582	0.0594	0.0606	0.0618	0.0630	0.0643	0.0655	0.0668
-1.4	0.0681	0.0694	0.0708	0.0721	0.0735	0.0749	0.0764	0.0778	0.0793	0.0808
-1.3	0.0823	0.0838	0.0853	0.0869	0.0885	0.0901	0.0918	0.0934	0.0951	0.0968
-1.2	0.0985	0.1003	0.1020	0.1038	0.1056	0.1075	0.1093	0.1112	0.1131	0.1151
-1.1	0.1170	0.1190	0.1210	0.1230	0.1251	0.1271	0.1292	0.1314	0.1335	0.1357
-1.0	0.1379	0.1401	0.1423	0.1446	0.1469	0.1492	0.1515	0.1539	0.1562	0.1587
-0.9	0.1611	0.1635	0.1660	0.1685	0.1711	0.1736	0.1762	0.1788	0.1814	0.1841
-0.8	0.1867	0.1894	0.1922	0.1949	0.1977	0.2005	0.2033	0.2061	0.2090	0.2119
-0.7	0.2148	0.2177	0.2206	0.2236	0.2266	0.2296	0.2327	0.2358	0.2389	0.2420
-0.6	0.2451	0.2483	0.2514	0.2546	0.2578	0.2611	0.2643	0.2676	0.2709	0.2743
-0.5	0.2776	0.2810	0.2843	0.2877	0.2912	0.2946	0.2981	0.3015	0.3050	0.3085
-0.4	0.3121	0.3156	0.3192	0.3228	0.3264	0.3300	0.3336	0.3372	0.3409	0.3446
-0.3	0.3483	0.3520	0.3557	0.3594	0.3632	0.3669	0.3707	0.3745	0.3783	0.3821
-0.2	0.3859	0.3897	0.3936	0.3974	0.4013	0.4052	0.4090	0.4129	0.4168	0.4207
-0.1	0.4247	0.4286	0.4325	0.4364	0.4404	0.4443	0.4483	0.4522	0.4562	0.4602
0.0	0.4641	0.4681	0.4721	0.4761	0.4801	0.4840	0.4880	0.4920	0.4960	0.5000

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986

Table 3: This table contains values of $\Pr(Z < z)$, where $Z \sim N(0, 1)$

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