Practical 6: Integral estimation

In this practical we will use Monte-Carlo techniques to estimate the area under a curve, i.e. we will perform *numerical integration*. It will help if you have your lecture notes for chapter 8 in front of you. As an example, we will try to estimate the following integral:

$$f(x) = abx^{a-1}(1-x^a)^{b-1} \quad \text{for} \quad 0 \le x \le 1\,,$$

where $a > 0$ and $b > 0$.

In the following example, we will use the values $a = 2.5$ and $b = 4$; then, we will plot the function. Type into R the following code:

```
a = 2.5; b = 4;
EvalF = function(x, a, b) {
    f = a*b*x^(a-1)*(1-x^a)^(b-1)
    return(f)
}
x = seq(0, 1, by=0.01)
y = EvalF(x, a, b)
plot(x, y, type="l")
```

To show the region which we're going to throw random numbers at (see chapter 8 lecture notes), we use the **abline** commands:

Listing 1: Rejection region

```
1 x = seq(0, 1, by=0.1)
2 y = EvalF(x, a, b)
3 plot(x, y, type="l", xlim=range(-1, 2))
4 abline(h=0, col=2, lty=2)
5 abline(h=2, col=2, lty=2)
6 abline(v=0, col=2, lty=2)
7 abline(v=1, col=2, lty=2)
```

Complete: What is the area of the rectangle that we throw random numbers at? _____

Complete: If we threw 1000 random numbers at the rectangle, and 500 landed under the curve, what is our estimate of the area under the curve? _____

To estimate the integral

$$\int_0^1 abx^{a-1}(1-x^a)^{b-1}\,dx$$

we can use the following function. Type this carefully into R, and then use it to approximate the integral given at the start of this handout.

```
Area = function(n, a, b) {
  no_of_hits = 0
  for(i in 1:n) {
```

```
    #Generate a value on the x-axis
    x = runif(1, 0, 1)
    #Generate a value on the y-axis
    y = runif(1, 0, 2)
    if(EvalF(x, a, b) > y) {#i.e. does the value land under the curve?
      no_of_hits = no_of_hits + 1
    }
  }
  area_of_rectangle = 2
  area_under_curve = area_of_rectangle * no_of_hits/n
  return(area_under_curve)
}
```

Assignment

Each person will have their own values of a and b. To generate your values, type the following:

```
set.seed(LOGIN_ID)
a = sample(3:14, 1); b = sample(3:14, 1)
```

where `LOGIN_ID` is your computer id without the 'a'.

1. For your values of a and b, estimate the area under the curve of $f(x)$, for $0 \leq x \leq 1$;
   - Produce a plot similar to that produced by listing 1, but alter your rejection region accordingly. What is your new area? Make $n = 10^6$ for your estimate.
   - Hand in a single page, with the plot above. State clearly your values of a, b and your estimate for the area under the curve.

2. A special function that comes up a lot in statistics is the standard Normal distribution, that is, $Z \sim N(0,1)$ – giving a bell-shaped curve with mean 0 and variance 1. Z has the following probability density function:
$$g(z) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{z^2}{2}\right).$$
Using Monte-Carlo integration, estimate the following integral:
$$\int_{-d}^{d} g(z)dz$$
where d is the value:
```
set.seed(LOGIN_ID)
d = round(runif(1, 0.5, 2.5), 2)
```
   - Produce a plot similar to Listing 1, but alter your rejection region accordingly. What is your new area?
   - For this question hand in two pages:
     - Page 1: A plot showing the sampling region and your function. State clearly your value of d and your integral estimate. Make sure $n = 10^5$ or more.
     - Page 2: Nicely formatted computer code for this question.

Your assignment should only be three pages long.