Learning outcomes: Chapters 6–8

1. You should understand the term *pseudo random number*, and you should be able to describe various methods for generating such numbers.

2. You should understand how to use a *congruential random number generator* of the form

$$r_i = (ar_{i-1} + b) \bmod m, \quad i = 1, \ldots, m.$$

3. You should know that the maximum period of a congruential random number generator is attained when:

(i) b and m have no common factors other than 1;

(ii) $(a-1)$ is a multiple of every prime that divides m;

(iii) $(a-1)$ is a multiple of 4 if m is a multiple of 4.

You should be able to use these criteria to check to see if the maximum period for a congruential generator is achieved.

4. You should memorise the standard R functions for random generation from some statistical distributions, including `runif` (uniform); `rbern` (Bernoulli); `rbinom` (Binomial); `rgeom` (geometric); `rpois` (Poisson); `rnorm` (Normal).

5, You should know how to use the `sample` command in R for drawing random samples.

6. You should know what the `set.seed` command does.

7. Given some values u_i simulated randomly from a Uniform $U(0,1)$ distribution, you should be able to obtain random observations from

- a given discrete probability distribution;
- a Bernoulli distribution;
- a Poisson distribution with known rate;
- a binomial distribution with known success probability.

For the above, it would help to memorise the standard probability functions for the binomial and Poisson distributions; for example, if $X \sim Poisson(\lambda)$, then

$$P(X = r) = \frac{e^{-\lambda}\lambda^r}{r!};$$

if $X \sim Bin(n, p)$, then

$$P(X = r) = \binom{n}{r} p^r (1-p)^{n-r}.$$

8. Given some values u_i simulated randomly from a Uniform $U(0,1)$ distribution, you should know that

$$x_i = 1 + \left\lfloor \frac{\log(1-u_i)}{\log(1-p)} \right\rfloor$$

gives values x_i from a geometric distribution with success probability p. Further, you should be able to *prove* this result.

9. In a general sense, you should know what *Monte Carlo* methods are, and you should be able to describe some basic simulation studies.

10. You should know how to approximate integrals using *Monte Carlo integration*, and you should be able to interpret R code which performs such numerical integration.