NEWCASTLE UNIVERSITY

SCHOOL OF MATHEMATICS & STATISTICS

SEMESTER 2 Paper 3

MAS1343

Computational Probability and Statistics

Time allowed: 1 hour 30 minutes

Candidates should attempt all questions. Marks for each question are indicated. However you are advised that marks indicate the relative weight of individual questions, they do not correspond directly to marks on the University scale.

There are SEVEN questions on this paper.

Answers to questions should be entered directly on this question paper in the spaces provided. Rough work should be done on the blank sides of the pages, or in the blank pages at the end of the paper. The rough work will not be marked. This question paper must be handed in, attached inside an anonymised cover sheet, at the end of the examination.

Calculators may be used. Statistical tables will be provided.

1. The following data set consist of temperature measurements across Europe on the 1st December, 2010.

```
-13.29, -5.01, -4.92, -3.76, -3.73, -1.83, -1.10
1.98, 2.34, 2.92, 3.45, 4.43, 6.75, 9.07,  9.57
```

(a) Calculate the sample median

Answer:

(b) Calculate first and third sample quartiles

Answer:

(c) Sketch a box-plot

Answer:

[**12 marks**]

2. Consider the following congruential generator:

$$r_i = (11 r_{i-1} + 9) \bmod 10 \quad (i = 1, 2, 3, \ldots),$$

with $r_0 = 9$.

(a) Is the maximum period for generators with modulo equal to 10 achieved for this generator? Give your reasons.

Answer:

(b) Not including the seed, generate and write down the first 10 terms of the sequence of integers given by this generator.

Answer:

Question 2 continued on next page

(c) What is the period (cycle length) of this generator with this seed?

Answer:

(d) By considering the sequence and its period, write down the value of the term r_{20002}.

Answer:

[**18 marks**]

3. The following set of random numbers $u_i; i = 1, \ldots, 4$ are four independent observations from a $U(0,1)$ distribution:

$$0.079 \quad 0.637 \quad 0.906 \quad 0.314$$

(a) Use the four given values of u_i to generate four observations from the Geometric random variable $G \sim \text{Geom}(0.5)$.

Answer:

(b) Now use the four given values of u_i to generate four observations from the Bernoulli random variable $B \sim \text{Bern}(0.6)$, stating the rule you are employing to achieve this.

Answer:

Question 3 continued on next page

(c) A Poisson random variable X has rate $\lambda = 1.2$. To three decimal places, the probability mass function is given by the table below. Use the four given values of u_i to generate four observations from X.

x	0	1	2	3	4	5	6
$\Pr(X = x)$	0.301	0.361	0.217	0.087	0.026	0.006	0.001

Answer:

[**20 marks**]

4. Consider the following R function:

```
f = function(x) {
n = length(x)
x = sort(x)
if(n %%  2 == 0) {
m = (x[n/2] + x[n/2 + 1])/2
} else {
m = x[(n+1)/2]
}
return(m)
}
```

(a) What does the above function do?

Answer:

(b) Give two ways that you would improve the function

Answer:

(c) Write an R function that takes in two vectors `x` and `y` and returns the ratio of their means. You may assume that the mean of each vector is positive.

Answer:

[**15 marks**]

5. Suppose that `x` is vector of doubles. Write down the R code that will:

(a) Calculate the length of `x`.

Answer:

(b) Select values of x that are greater than 50.

Answer:

(c) That will add 10 to each element of `x`.

Answer:

(d) That will calculate the median of `x`.

Answer:

(e) Select values of x that are greater 5 but less than 8.

Answer:

Hint: Each question only requires a single line of R code.

[**10 marks**]

6. (a) K is a non-negative real-valued integrable. For K to be considered a suitable kernel, what two requirements should it fulfil.

Answer:

(b) Using the Uniform Kernel

$$K(t) = \begin{cases} \frac{1}{\sqrt{3}} & -\sqrt{3} < t < \sqrt{3} \\ 0 & \text{otherwise.} \end{cases}$$

Sketch a density plot for the following 3 points

$$0, 0.5, 0.75$$

Answer:

Question 6 continued on next page

[**15 marks**]

7. Write down a short piece of R code that would perform a Monte-Carlo estimate of

$$\int_0^1 x^3 dx\,.$$

Answer:

[**10 marks**]

THE END

For Rough Work Only