NEWCASTLE UNIVERSITY

SCHOOL OF MATHEMATICS & STATISTICS

SEMESTER 2 Paper 2

MAS1343

Computational Probability and Statistics

Time allowed: 1 hour 30 minutes

Candidates should attempt all questions. Marks for each question are indicated. However you are advised that marks indicate the relative weight of individual questions, they do not correspond directly to marks on the University scale.

There are EIGHT questions on this paper.

Answers to questions should be entered directly on this question paper in the spaces provided. Rough work should be done on the blank sides of the pages, or in the blank pages at the end of the paper. The rough work will not be marked. This question paper must be handed in, attached inside an anonymised cover sheet, at the end of the examination.

Calculators may be used. Statistical tables will be provided.

1. For the data set:

$$1, \quad 8, \quad -10, \quad 3, \quad 9, \quad -7, \quad 20, \quad -8, \quad 4, \quad 0, \quad 8.$$

find the

(a) the sample mean

Answer:

(b) the sample median

Answer:

(c) the sample mode

Answer:

(d) the sample standard deviation s

Answer:

(e) the sample range

Answer:

[15 marks]

2. The sample variance of a set of n observations $x_1, x_2, \ldots, x_n$ is usually defined as:

$$\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2.$$

Show that this can be re-expressed as

$$\frac{1}{n-1}\left(\sum_{i=1}^{n} x_i^2 - n\bar{x}^2\right).$$

Answer:

[10 marks]

3. (a) Given an observation u from a $U(0,1)$ distribution, write down an expression for an observation x from a $Geometric(p)$ distribution.

Answer:

(b) Use the following two numbers simulated from a $U(0,1)$ distribution to generate two observations from a $Geometric(0.1)$ distribution:

$$0.643, \quad 0.954.$$

Answer:

[10 marks]

4. (a) Consider a congruential generator of form:

$$r_i = (ar_{i-1} + b) \bmod m \quad (i = 1, 2, 3, \ldots),$$

with seed r_0, multiplier a, additive constant $b > 0$, and modulo m. State the maximum possible period of such a generator, and the conditions under which this will be achieved.

Answer:

(b) Consider the following congruential generator:

$$r_i = (11r_{i-1} + 2) \bmod 7 \quad (i = 1, 2, 3, \ldots),$$

with $r_0 = 5$.

(i) Not including the seed, write down the first 10 terms of this generator.

Answer:

Question 4 continued on next page

(ii) What is the period (cycle length) of this generator with this seed?
Answer:

(iii) By considering the sequence and its period, write down the value of the term r_{20002}.
Answer:

[20 marks]

5. You have been given an unbiased 10 sided-die. Using the following uniform random number, simulate four values from this die.

0.907, 0.792, 0.050, 0.125,

Show your reasoning.

Answer:

[8 marks]

6. Suppose that `dd` is a data frame with three columns: `a`, `b` and `c`.

(a) What does the command `dim(dd)` do?

Answer:

(b) What data type does `dim(dd)` return?

Answer:

(c) What does the command:

```
dd[dd$a > 10 | dd$b < 0,]
```

do?

Answer:

(d) Briefly explain why naming a column `c` may be poor choice.

Answer:

[9 marks]

7. (a) Give example R code that creates

(i) A boolean

Answer:

(ii) A data frame

Answer:

(iii) A vector

Answer:

(iv) A double

Answer:

Question 7 continued on next page

(b) The following piece of R code contains a bug. Clearly highlight the bug and suggest a possible fix.

```
x = 0
for(i in 5:-5) {
  if (i <<= 1) {
    x = x + i
  }
}
y = x
```

Answer:

[13 marks]

8. Consider the piece of R code.

```
set.seed(1)
x = runif(10)
no_of_hits = 0
for(i in 1:5) {
  if(x[i]^2 > x[5+i]) {
    no_of_hits = no_of_hits + 1
  }
}
area_estimate = no_of_hits/5
```

(a) Briefly describe what the above piece of R is trying to do.

Answer:

(b) Given

```
> set.seed(1)
> round(runif(10), 2)
 [1] 0.27 0.37 0.57 0.91 0.20 0.90 0.94 0.66 0.63 0.06
```

Calculate the value of `area_estimate`.

Answer:

[15 marks]

THE END

The Standard Normal Distribution

z	-0.09	-0.08	-0.07	-0.06	-0.05	-0.04	-0.03	-0.02	-0.01	-0.00
-2.9	0.0014	0.0014	0.0015	0.0015	0.0016	0.0016	0.0017	0.0018	0.0018	0.0019
-2.8	0.0019	0.0020	0.0021	0.0021	0.0022	0.0023	0.0023	0.0024	0.0025	0.0026
-2.7	0.0026	0.0027	0.0028	0.0029	0.0030	0.0031	0.0032	0.0033	0.0034	0.0035
-2.6	0.0036	0.0037	0.0038	0.0039	0.0040	0.0041	0.0043	0.0044	0.0045	0.0047
-2.5	0.0048	0.0049	0.0051	0.0052	0.0054	0.0055	0.0057	0.0059	0.0060	0.0062
-2.4	0.0064	0.0066	0.0068	0.0069	0.0071	0.0073	0.0075	0.0078	0.0080	0.0082
-2.3	0.0084	0.0087	0.0089	0.0091	0.0094	0.0096	0.0099	0.0102	0.0104	0.0107
-2.2	0.0110	0.0113	0.0116	0.0119	0.0122	0.0125	0.0129	0.0132	0.0136	0.0139
-2.1	0.0143	0.0146	0.0150	0.0154	0.0158	0.0162	0.0166	0.0170	0.0174	0.0179
-2.0	0.0183	0.0188	0.0192	0.0197	0.0202	0.0207	0.0212	0.0217	0.0222	0.0228
-1.9	0.0233	0.0239	0.0244	0.0250	0.0256	0.0262	0.0268	0.0274	0.0281	0.0287
-1.8	0.0294	0.0301	0.0307	0.0314	0.0322	0.0329	0.0336	0.0344	0.0351	0.0359
-1.7	0.0367	0.0375	0.0384	0.0392	0.0401	0.0409	0.0418	0.0427	0.0436	0.0446
-1.6	0.0455	0.0465	0.0475	0.0485	0.0495	0.0505	0.0516	0.0526	0.0537	0.0548
-1.5	0.0559	0.0571	0.0582	0.0594	0.0606	0.0618	0.0630	0.0643	0.0655	0.0668
-1.4	0.0681	0.0694	0.0708	0.0721	0.0735	0.0749	0.0764	0.0778	0.0793	0.0808
-1.3	0.0823	0.0838	0.0853	0.0869	0.0885	0.0901	0.0918	0.0934	0.0951	0.0968
-1.2	0.0985	0.1003	0.1020	0.1038	0.1056	0.1075	0.1093	0.1112	0.1131	0.1151
-1.1	0.1170	0.1190	0.1210	0.1230	0.1251	0.1271	0.1292	0.1314	0.1335	0.1357
-1.0	0.1379	0.1401	0.1423	0.1446	0.1469	0.1492	0.1515	0.1539	0.1562	0.1587
-0.9	0.1611	0.1635	0.1660	0.1685	0.1711	0.1736	0.1762	0.1788	0.1814	0.1841
-0.8	0.1867	0.1894	0.1922	0.1949	0.1977	0.2005	0.2033	0.2061	0.2090	0.2119
-0.7	0.2148	0.2177	0.2206	0.2236	0.2266	0.2296	0.2327	0.2358	0.2389	0.2420
-0.6	0.2451	0.2483	0.2514	0.2546	0.2578	0.2611	0.2643	0.2676	0.2709	0.2743
-0.5	0.2776	0.2810	0.2843	0.2877	0.2912	0.2946	0.2981	0.3015	0.3050	0.3085
-0.4	0.3121	0.3156	0.3192	0.3228	0.3264	0.3300	0.3336	0.3372	0.3409	0.3446
-0.3	0.3483	0.3520	0.3557	0.3594	0.3632	0.3669	0.3707	0.3745	0.3783	0.3821
-0.2	0.3859	0.3897	0.3936	0.3974	0.4013	0.4052	0.4090	0.4129	0.4168	0.4207
-0.1	0.4247	0.4286	0.4325	0.4364	0.4404	0.4443	0.4483	0.4522	0.4562	0.4602
0.0	0.4641	0.4681	0.4721	0.4761	0.4801	0.4840	0.4880	0.4920	0.4960	0.5000

Table 1: *This table contains values of* $P(Z \leq z)$ *where* $Z \sim N(0,1)$.

continued ...

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986

Table 2: *This table contains values of* $P(Z \geq z)$ *where* $Z \sim N(0,1)$.