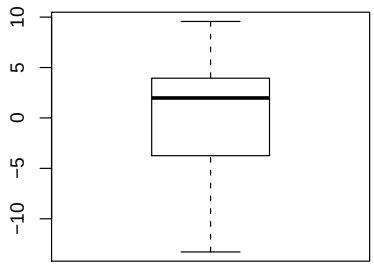
MAS1343 - Paper 3 solutions

Question 1

1. 1.98
2. -3.76, 4.43
3. Boxplot (remember to check for outliers)



Question 2

1. $a = 11$, $b = 9$, $m = 10$ and $r_0 = 9$. Hence
   (i) b and m have no common factors other than 1.
   (ii) $(a-1)$ is a multiple of every prime that divides m. True since $a - 1 = 10 = m$.
   (iii) $(a-1)$ is a multiple of 4 if m is a multiple of 4. Trivially true, since $m = 10$ is not a multiple of 4. Since conditions (i) (ii) and (iii) are satisfied, the maximum period for this choice of modulo is reached, and this period is $m = 10$.
2. $r_1 = (11 \times r_0 + 9) \mod 10 = 108 \mod 10 = 8$. This gives $8, 7, 6, 5, 4, 3, 2, 1, 0, 9, 8$
3. Period is 10
4. 7.

Question 3

1. $u_i = \{0.079, 0.637, 0.906, 0.314\}$. To simulate from a geometric distribution we use:

$$X = 1 + \left[\frac{\log(1-U)}{\log(1-p)}\right]$$

   giving 1,2,4,1
2. Rule: If $u_i > 0.6$, simulate a 1, else simulate a 0. This give 0, 1,1,0.
3. We need the cdf table, i.e.

x	0	1	2	3	4	5	6
$\Pr[X \leq x]$	0.301	0.662	0.879	0.966	0.992	0.998	0.999

So our random numbers are: 0, 1, 3, 1

Question 4

1. Works out the median
2. Indent the code, use a more descriptive function name, use brackets around the modulus term.
3.

```
g = function(x, y) {
  return(mean(x)/mean(y))
}
```

Question 5

1. `length(x)`
2. `x[x>50]`
3. `x+10`
4. `median(x)`
5. `x[x>5 & x<8]`

Question 6

1. The two requirements are:
$$\int_{-\infty}^{\infty} K(t)dt = 1$$
and
$$K(-t) = K(t)$$
for all values of t.
2. A sketch similar to Figure 9.2(d) in the notes. Except with the uniform Kernel.

Question 7

This is a **hard** question.

```
N = 10000

hit = 0
for(i in 1:N) {
  if(runif(1)^3 > runif(1))
    hit = hit + 1
```

```
}
hit/N

#Or
x = runif(N); y = runif(N)
sum(x^3 > y)/N
```