

MAS1343 - Specimen Paper 1 Solutions

Warning: there may be typos in these solutions. When working through the solutions check your versions with the one on BB (see the date in the header)

Question 1

1. Multiple modes: 68, 72, 76, 80
2. 72
3. 68, 80

Question 2

$$\begin{aligned}
 \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2 &= \frac{1}{n-1} \sum_{i=1}^n (x_i^2 - 2\bar{x}x_i + \bar{x}^2) \\
 &= \frac{1}{n-1} \left[\sum_{i=1}^n x_i^2 - 2 \sum_{i=1}^n \bar{x}x_i + \sum_{i=1}^n \bar{x}^2 \right] \\
 &= \frac{1}{n-1} \left[\sum_{i=1}^n x_i^2 - 2n\bar{x}^2 + n\bar{x}^2 \right] \\
 &= \frac{1}{n-1} \left(\sum_{i=1}^n x_i^2 - n\bar{x}^2 \right).
 \end{aligned}$$

Question 3

1. 7
2. No, since $7-1 = 6$ which is not a multiple of all factors of $m = 10$
3. 8,9,6
4. Divide r_i by $m - 1$.

Question 4

1. Calculates the length of a **vector**
2. Reverses the order of a vector
3. Tabulates the values of a vector
4. Gives the dimensions of a data frame
5. Creates a data frame

Question 5

We need to work out the CDF for this distribution:

$$\Pr(Z \leq z) = \sum_{i=2}^z \binom{z-1}{1} 0.5^2 \times 0.5^{z-2}$$

which gives:

z	2	3	4	5	6	7	8	9
$\Pr(Z \leq z)$	0.25	0.50	0.69	0.81	0.89	0.94	0.96	0.98

The our random numbers from this distribution are: 2, 4, 7

Question 6

1. == tests for equality and = is for assignment.
2. 16
3. .

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if (y>0){
  z = z + 1
}
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4. String (or character), vector, data frame

Question 7

If $u < 0.75$ we simulate a head, otherwise a tail. This gives:

T, T, H, H

Question 8

1. From the question we have

$$f(x) = 8x(1-x^2)^3 \quad \text{for } 0 \leq x \leq 1$$

We can either use the graph on the exam paper to decide if the (x, y) points lie below the curve or we can construct a table of values:

x	0.32	0.34	0.63
$f(x)$	1.85	1.88	
y	0.56	1.77	
$f(x) > y$	TRUE	TRUE	

From the table we see that:

$$\int_0^1 f(x) dx \simeq \frac{4}{10} \times \text{Area of rectangle} = 0.8$$

x	0.32	0.34	0.63	0.11	0.08	0.90	0.07	0.91	0.67	0.68
$f(x)$	1.85	1.88	1.11	0.85	0.63	0.05	0.55	0.04	0.90	0.85
y	0.56	1.77	0.35	0.78	0.88	1.42	0.85	0.10	1.09	1.22
$f(x) > y$	TRUE	TRUE	TRUE	TRUE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE