1. (a) Intercept – 400 – fixed costs/overheads.
Gradient – 30 – cost per part.

(b) $30x + 400 < 900$
$\Rightarrow x < 16.6667$
$\Rightarrow$ 16 parts or less.

(c) $P = 50x - (30x + 400)$
$= 20(x - 20)$, so $A = 20$.

(d) $P = -250 - 30x + 230x^{1/2}$
$\Rightarrow \frac{dP}{dx} = -30 + 115x^{-1/2}$ (and $\frac{d^2P}{dx^2} = -\frac{115}{2}x^{-3/2}$)

For Turning Points, $\frac{dP}{dx} = 0$

$\Rightarrow \frac{115}{\sqrt{x}} = 30$; $30\sqrt{x} = 115$; $x = \left(\frac{115}{30}\right)^2 = 14.694$.

When $x = 14.694$, $\frac{d^2P}{dx^2} < 0 \therefore$ Maximum T.P.

So produce $x = 15$ parts, giving $P \cong$ £191 per day.

2. (a) $\frac{dy}{dx} = x^2 + 4x - 12$ (and $\frac{d^2y}{dx^2} = 2x + 4$)
For Turning Points, $\frac{dy}{dx} = 0$
$\Rightarrow (x+6)(x-2) = 0$; $x = 2$ or $x = -6$.
($y = \frac{-25}{3}$) ($y = 77$)

When $x = 2$, $\frac{d^2y}{dx^2} > 0 \therefore$ Maximum T.P; similarly, $x = -6$ Min. T.P.

(b) $\frac{\partial f}{\partial p} = 12p^2q + 6q^2$
$\frac{\partial f}{\partial q} = 4p^3 + 12pq$.

3. $y = 30x + 300$.

(a) Intercept – 300 – fixed costs of £300,000.
Gradient – 30 – cost of £30,000 per part.

(b) $30x + 300 < 1000$

$\Rightarrow x < 23.\dot{3}\dot{3}$

so $(12, \ldots, 23)$

(c) $52x - (30x + 300) - 100$

$52x - 30x - 300 - 100$

so $\pi = 22x - 400$.

(d) $\frac{\partial \pi}{\partial x} = -x^2 + 2x + 360$ (and $\frac{\partial^2 \pi}{\partial x^2} = -2x + 2$)

For Turning Points, $\frac{\partial \pi}{\partial x} = 0$.

$\Rightarrow -x^2 + 2x + 360 = 0$.

$D = 2^2 - 4 \cdot -1 . 360 = 1444$.

so $x = \dfrac{-2 \pm \sqrt{1444}}{-2} \Rightarrow x = 20$ or $x = -18$.

x cannot be -ve, so $x = 20$. $\Rightarrow \pi \cong$ £3.9 million.

(Also, when $x = 20$, $\frac{\partial^2 \pi}{\partial x^2} < 0$, $\therefore$ Maximum T.P.).

4. (a) Maximise $P = 20x + 25y$ subject to:

$2x + 4y \leq 50$
$3x + y \leq 25$
$x, y \geq 3$.

1. (a) $$\bar{x}_F \approx \frac{29\times3 + 31\times15 + 33\times27 + \dots + 39\times16}{3 + 15 + \dots + 16} = £34,933.$$

Approximation as we use the midpoints.

(b). Male data seems positively skewed. Use the median here?

(c) $P(\text{salary} > £36{,}000 \mid \text{salary} > £30{,}000)$

$$= \frac{10+29+6+16+2+1}{26+15+19+27+16+60+10+29+6+16+2+1}$$

$$= \frac{64}{227}.$$

(d) see graph.

2. $\bar{x} = 68{\cdot}1\%$; $s = 10{\cdot}1\%$.

4. (a) $$s^2 = \frac{\sum x^2 - n\bar{x}^2}{n-1} \simeq \frac{\sum fx^2 - n\bar{x}^2}{n-1}.$$

$$\sum fx^2 = 11\times25^2 + 25\times27^2 + \dots + 1\times55^2 = 160636.$$

$$\Rightarrow s^2 \cong \frac{160636 - 164\times31^2}{163} = 18{\cdot}601.$$

Approximation as we use mid-points.

(b) Females will be smaller as these data are clearly less dispersed.

(c). IQR is not distorted by outliers or skewed data. The range, however, is.

1. X : No. of customers put 'on hold'.

(a) $X \sim Bin(15, 0.3)$

Fixed no. of trials, each one having two outcomes.

(b) $E[X] = 4.5$ calls; $SD(X) = 1.77$ calls.

(c) $P(X<2) = 0.3527$.

(d) (i) $T \sim Exponential(\lambda)$.

$E[T] = \frac{1}{\lambda} = \frac{1}{2} \Rightarrow \lambda = 2$.

so $T \sim Exp(2)$.

(ii) $P(T < \frac{1}{4}) = 1 - e^{-2.(1/4)} = 0.393$.

(iii) $1 - e^{-\lambda(1/4)} = \frac{1}{4}$

$\Rightarrow \lambda = 1.15$, NOT 2!

Chapter 5.

2. (a) Constant rate.

(b) (i) $E[X] = 4.5$ records; $SD(X) = \sqrt{4.5} = 2.12$ records.

(ii) $P(X<2) = 0.0611$.

(c) Y: No. of companies fined.

$Y \sim Bin(6, 0.9389)$

$P(Y>4) = P(Y=5) + P(Y=6) = 0.9525$

3. (a) $X \sim Poisson(4)$.

(b) $P(1 \leq X \leq 3) = P(X=1) + P(X=2) + P(X=3) = 0.415$

(c) $P(T > 0.5) = 1 - P(T < 0.5)$

$= 1 - \{1 - e^{-4 \times 0.5}\} = 0.135$.

Chapter 5.

CHAPTER 5

1. $X \sim N(350, 98.5^2)$

(a) $P(X > 450) = P(Z > 1.02) = 1 - 0.8461 = 0.1539$

(b) $P(350 < X < 450) = P(Z < 1.02) - P(Z < 0) = 0.8461 - 0.5 = 0.3461$

(c) From tables, $P(Z > 1.96) = 2.5\%$

$$\Rightarrow 1.96 = \frac{X - 350}{98.5} \Rightarrow X = £543.06$$

2. $X \sim N(31, 2.3^2)$

(a) $P(X < 26) = P(Z < -2.17) = 0.015$

(b) $P(X > 37) = P(Z > 2.61) = 1 - 0.9955 = 0.0045$

(c) We want $P(X > LQ) = 0.75$

OR $P(X < LQ) = 0.25$

From tables, $P(Z < -0.67) \approx 0.25$

$$\Rightarrow -0.67 \approx \frac{LQ - 31}{2.3} \Rightarrow LQ \approx £29{,}459$$

CHAPTER 6

1. (a) $10.4 \pm 2.093 \times \frac{3.25}{\sqrt{20}} \longrightarrow £(8.88, 11.92)$

(b) Narrower, as we are less confident.

(c) $H_0: \mu_S = \mu_B$ versus $H_1: \mu_S \neq \mu_B$

$$s = \sqrt{\frac{19 \times 3.25^2 + 24 \times 2.2^2}{}} = 2.7145$$

$\Rightarrow t = 2.64$ $p < 1\%$

From tables:

p	10%	5%	1%
critical value	1.645	1.96	2.576

* Strong evidence against H_0.
* Reject H_0, accept H_1.
* There is evidence to suggest a real difference in spends at the two shops.

2. $H_0: \mu = 350$ versus $H_1: \mu > 350$

$t = 1.777$ — p between 1% & 5%

From tables:

p	10%	5%	1%
	1.318	1.711	2.492

* Moderate evidence against H_0.
* Reject H_0 in favour of H_1.
* There is evidence to support the newspaper's claim.

CHAPTER 7.

1. $S_{xy} = -475.8$, $S_{xx} = 2198$, $S_{yy} = 175.425$.

(a) $r = -0.679 \Rightarrow$ moderate negative linear association.

(b) $\hat{\beta}_1 = \dfrac{-475.8}{2198} = -0.216$; $\hat{\beta}_0 = 11.65 + 0.216 \times 37 = 19.642$.

$$\Rightarrow y = 19.642 - 0.216x + \varepsilon, \quad \varepsilon \sim N(0, \sigma^2).$$

slope $= -0.216$: with every 1 year increase in age, the predicted number of blips will decrease by 0.216.

(c) For J. King, $y = 8.842 < 9$, so <u>No</u>, we cannot expect him to pass the test!

2. (a) The relationship is significant, as the p-value for the test $H_0: \rho = 0$ is less than 0.05 and so we would reject H_0 here. There is a significant, positive linear association.

(b) $S_{xy} = 348.815$; $S_{xx} = 566.521$; $S_{yy} = 254.949$

$\Rightarrow y = 0.349 + 0.616x + \varepsilon, \ \varepsilon \sim N(0, \sigma^2)$

$\Rightarrow$ when $x = -2.5$, $y = -1.191$.