Chapters 1 and 2

Question 1

EZY Electricals produce parts for large plasma televisions. They currently have a contract to produce a part for a batch of Samsung's latest 3D televisions.

(a) One of their analysts suggests the following daily linear cost function, where total costs for this contract £y are related to x, the number of parts required:

$$y = 30x + 400.$$

In plain English, give a practical interpretation of the intercept and the gradient.

(b) Using this linear cost function, at most how many parts should be produced if costs are to be kept below £900 per day?

(c) If Samsung pay *EZY Electricals* £50 per part produced, find a function for the daily profit P in the form

$$P = A(x - A),$$

assuming the linear cost function from part (a), where A is to be found.

(d) After re-negotiating the terms of their agreement with Samsung, the company are able to re-express their profit function according to the following non-linear profit function:

$$P = -250 - 30x + 230\sqrt{x}.$$

Using differentiation, optimise this problem to find the number of parts the company should produce to maximise their daily profit. What is the maximum daily profit?

Question 2

(a) Find the stationary points on the curve $y = \frac{1}{3}x^3 + 2x^2 - 12x + 5$, and determine their nature.

(b) Find the first-order partial derivatives of the following function of p and q :

$$f(p, q) = 4p^3q + 6pq^2 - 10.$$

Question 3

Sabre Airline Solutions (SAS) produce parts for Boeing's new 787 Dreamliner aircraft. An analyst working at SAS suggests the following weekly linear cost function, where total costs for a current contract, £y (thousand), are related to x, the number of parts produced:

$$y = 30(x + 10).$$

(a) Give a practical interpretation of the intercept and gradient.

(b) Boeing require at least 12 parts from SAS every week. If SAS aim to keep weekly costs below £1 million, find a range of suitable values for the number of pats they should produce each week.

(c) Suppose that, each week, we assume

$$\text{Profit} = \text{Income} - \text{costs}.$$

Boeing pay SAS £52,000 for each part produced. They also pay a fixed bonus of £100,000 if they produce the minimum of 12 parts per week, but apply a penalty of £100,000 if SAS do not meet the minimum requirement of 12 parts per week. Find a linear function for the weekly profit, π, assuming the linear cost function from part (a), if SAS do *not* produce the minimum number of parts required by Boeing.

(d) Suppose the following profit function is deemed more realistic:

$$\pi = -\frac{1}{3}x^3 + x^2 + 360x - 1000.$$

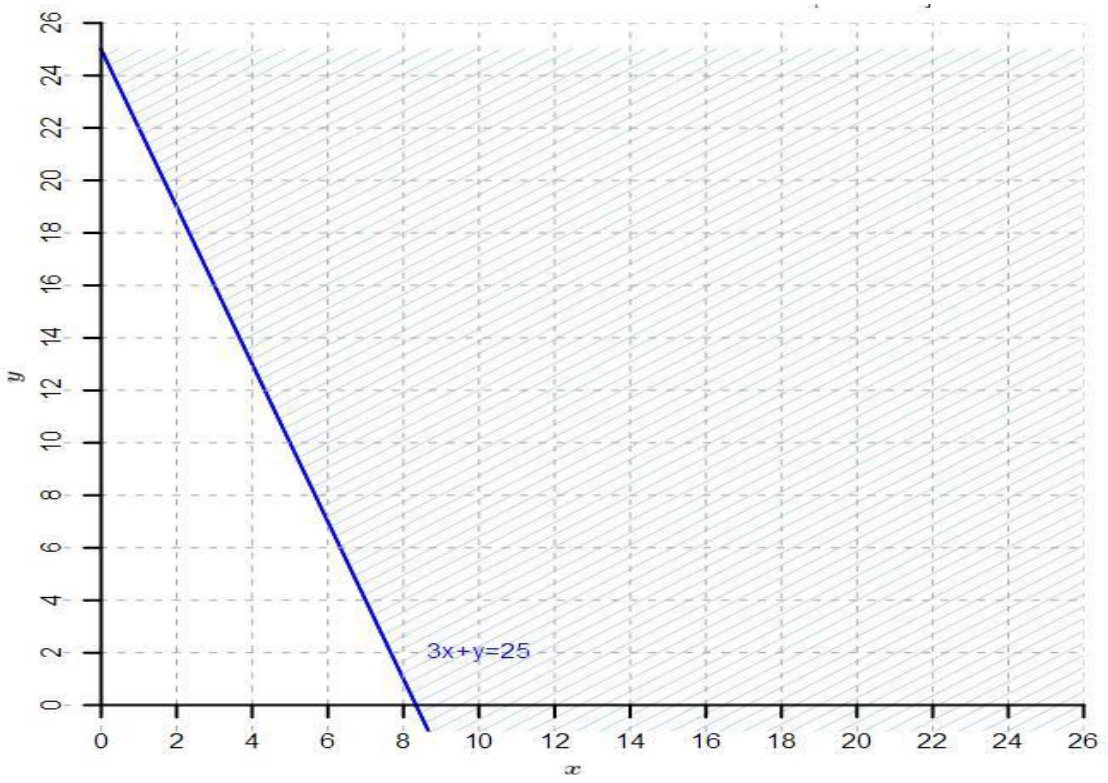
Show that profits will be maximised when the company produce 20 parts per week, and find the corresponding weekly profit.

Question 4

Tony TV produce standard digital televisions and 3D digital televisions. Each standard digital television requires 2 turbo–boards and 3 soundcards. Each 3D digital television requires 4 turbo–boards and just 1 soundcard. Each day, the company has 50 turbo–boards and 25 soundcards available. Current demand also requires the company to make at least 3 of each type of television each day. The company sells each standard digital television for a profit of £20 and each 3D digital television for a profit of £25. The company makes and sells *x* standard digital televisions and *y* 3D digital televisions each day.

(a) Formulate the company's situation as a linear programming problem, and complete the graph below.

(b) Using the graph in part (a), find the company's maximum daily profit.



Chapter 3

Question 1

Shown below is a grouped frequency table of the salaries of 314 randomly selected employees at *PricewaterhouseCoopers*.

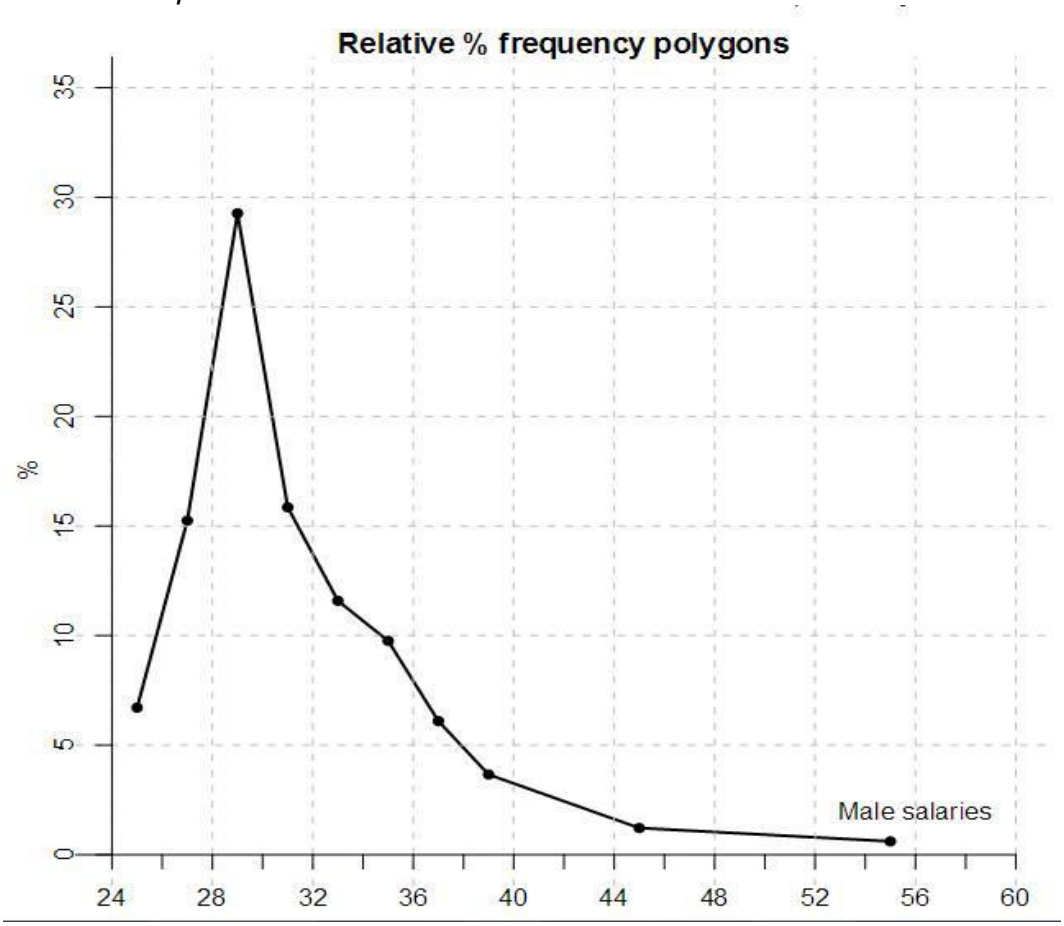
Salary (X thousand £)	Male	Female
$24 \leq x < 26$	11	0
$26 \leq x < 28$	25	0
$28 \leq x < 30$	48	3
$30 \leq x < 32$	26	15
$32 \leq x < 34$	19	27
$34 \leq x < 36$	16	60
$36 \leq x < 38$	10	29
$38 \leq x < 40$	6	16
$40 \leq x < 50$	2	0
$50 \leq x < 60$	1	0

(a) Use the table above to find the sample mean salary of female employees. Why is your answer here an *approximation*?

(b) The mean salary for males is £31,000. Why might this measure of average not be the most appropriate? Which measure would you recommend here?

(c) Using the frequency table, estimate the probability that a randomly chosen employee earning at least £30,000 has a salary of at least £36,000.

(d) Complete the graph below, and use this to compare male and female salaries at *PricewaterhouseCoopers*.



Question 2

The Net Promoter Score (NPS) is a web-based tool for companies to evaluate their customer loyalty. A major UK clothing retailer recently sampled seven of its online customer accounts, resulting in the following NPS ratings (out of 100):

81 62 73 50 66 70 75

Find the mean and standard deviation for this sample of Net Promoter Scores.

Question 3

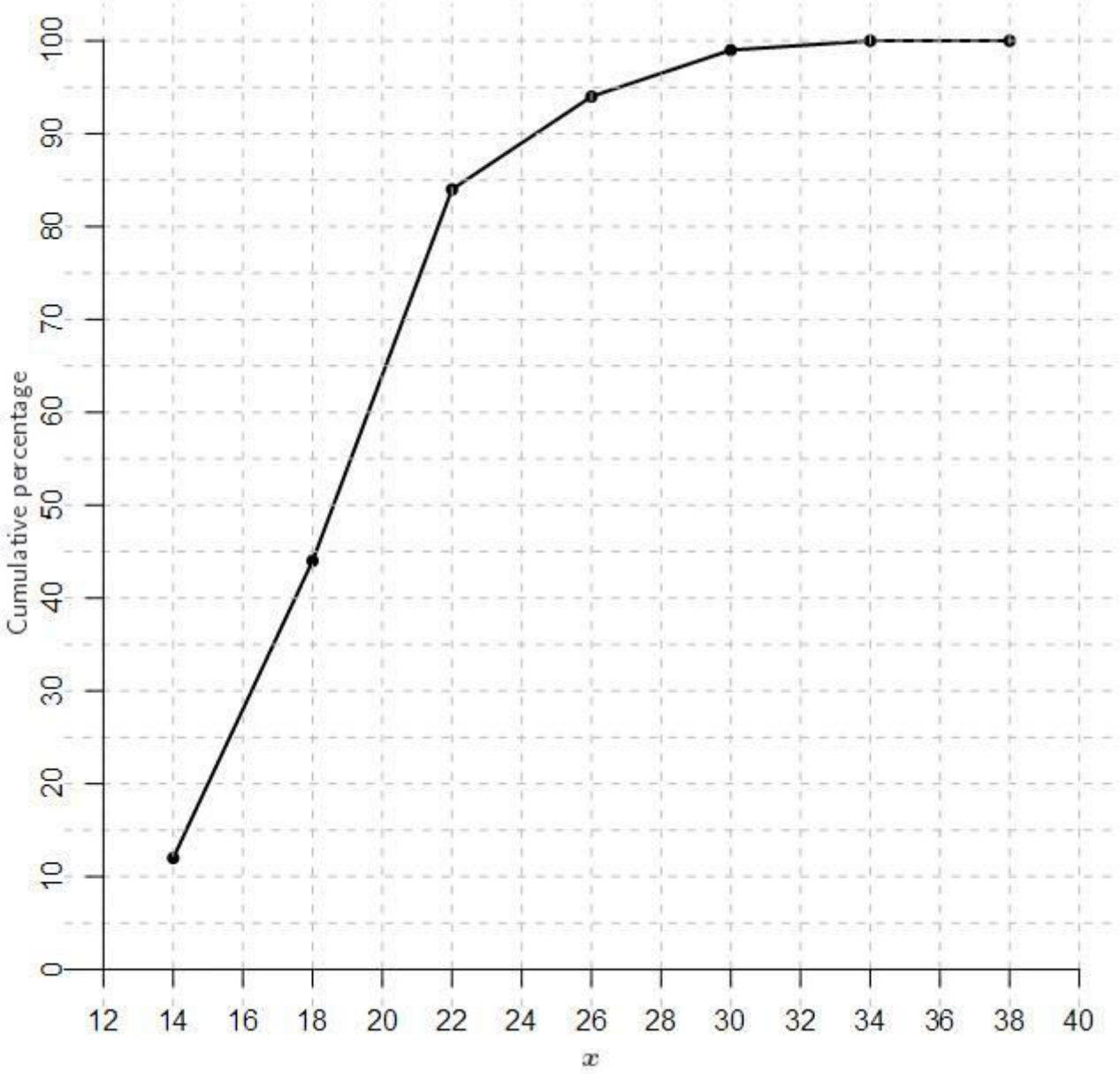
In October 2010 model and celebrity Kate Moss endorsed the latest Top Shop women's clothing range, where she starred in an advertising campaign aimed at increasing sales (see pictures below). Total sales of women's clothing at a selection of Top Shop stores are summarised in the table underneath, for periods before and after this advertising campaign.

Total sales (X thousand pounds)	Percentage		Cumulative percentage	
	Before	After	Before	After
$10 \leq x < 14$	12		12	2
$14 \leq x < 18$	32		44	17
$18 \leq x < 22$	40		84	
$22 \leq x < 26$	10	38	94	77
$26 \leq x < 30$	5		99	94
$30 \leq x < 34$	1	4	100	
$34 \leq x < 38$	0	2	100	

(a) Complete the table above.

(b) Complete the graph below, and estimate the median and inter-quartile range for sales in each period.

(c) Use your answers above to assess the effectiveness of the advertising campaign. Why must you be cautious about any conclusion you draw here?



Question 4

(a) Find the sample variance for male employees' salaries in the table in question 1. Why is this an approximation of the sample variance?

(b) Would you expect the sample variance for female employees' salaries to be smaller or larger than that for males? Explain your answer.

(c) Why might the inter-quartlie range be preferred to the standard deviation in some instances? Why is the range the least used measure of spread?

Chapter 4

Question 1

The Holiday Hypermarket call centre can receive, at most, fifteen calls in any one minute period (otherwise customers hear an "engaged" tone). Experience has shown that 30% of all customers who do not hear the engaged tone are put 'on hold' until an operator can take their call.

(a) What probability distribution might be reasonable to use to model the number of customers who are put 'on hold' in any one minute period? Justify your answer.
(b) Find the expected number, and standard deviation, for the number of calls put 'on hold' in any one minute period.
(c) Find the probability that fewer than two calls are put 'on hold' in any one minute period.
(d) Suppose that T represents the time between successful telephone sales at the Holiday Hypermarket.
    i. If we can usually expect to wait around half a minute between two successful telephone sales, propose an appropriate distribution for T.
    ii. Use you answer to part (i) to find the probability that the time between two consecutive successful telephone sales is less than 15 seconds.
    iii. Suppose we are told that the probability in part (ii) should be around one quarter. How would this affect the model you proposed in part (i)?

Question 2

When auditing a company's accounts, experience suggests that around 3 records in every 10 will be materially mis-stated.

(a) Let X be the number of records materially mis-stated in a random sample of 15 such records. What is the main assumption when using $X \sim \text{Poisson}(4.5)$?
(b) Assuming the distribution in part (a),
    i. find $E[X]$ and $SD(X)$;
    ii. find the probability that in a sample of 15 records, fewer than two are materially mos-stated.
(c) Auditors sample 15 records from 6 randomly chosen companies. The company is fined if at least two of their records are materially mis-stated. Using your answer to part (b)(ii), find the probability that more than four of these companies will be fined.

Question 3

An energy company offers an online chat service to customers using their website, to help them with their accounts. Past data suggests that customers start an online chat at a rate of 4 per minute.

(a) What would be an appropriate probability distribution for the number of online chats started in a one minute period?
(b) Find the probability that between one and three online chats are started in a one minute interval.
(c) Assume that the time between the start of online chats follows an exponential distribution. What proportion of the time is there more than 30 seconds between the start of online chats?

Chapter 5

Question 1

Assuming customers' bills for a weekend stay at a hotel are Normally distributed with mean £350 and standard deviation £98.50, find

(a) the probability that a randomly chosen bill exceeds £450;
(b) the probability that a randomly chosen bill is between £350 and £450;
(c) the size of a bill which is exceeded on 2.5% of occasions.

Question 2

The average salary at a company is £31,000 with a standard deviation of £2,300. Assuming salaries are Normally distributed, find

(a) the probability that the salary of a randomly chosen employee does not exceed £26,000;
(b) the probability that the salary of a randomly chosen employee is greater than £37,000;
(c) the lower quartile salary.

Chapter 6

Question 1

A discount supermarket has collected data on how much customers spend on fresh fruit and vegetables in a week. They sampled 20 randomly selected customers at their shop in South Shields and 25 randomly selected customers at their shop in Byker. The mean and standard deviation from each sample are given in the table below.

Supermarket	Mean	St. dev.
South Shields	£10.40	£3.25
Byker	£8.25	£2.20

(a) Find a 95% confidence interval for the mean amount spend on fresh fruit and vegetables in a week at the shop in South Shields.
(b) Without calculating it, would a 90% confidence interval be narrower or wider than that in part (a)?
(c) Test the null hypothesis that there is no difference between the average spend on fresh fruit and vegetables at the two shops in a week.

Question 2

The manager of a hotel claims that the average guest bill for a weekend is £350. A local journalist, however, believes that prices have increased and that average bills are greater than £350. To investigate, the journalist took a sample of 25 bills from customers who had recently stayed at the hotel; the mean from this sample was £385 with a standard deviation of £98.50.

The next evening, an article appeared in the local newspaper with the following headline:

Local Hotel More Expensive Than It Claims!

Perform an appropriate hypothesis test to see if the newspaper's headline can be supported.

Chapter 7

Question 1

Most supermarket chains assess the performance of staff working on the checkouts by monitoring the speed at which they use the electronic scanners. For a random sample of ten employees at a particular branch of Morrisons, the age of each employee (*x*, in years) and the average number of transactions processed per minute over a two hour test period (*y*, blips), is recorded for 10 employees. The following summaries were obtained:

$$\sum_{i=1}^{10} x_i = 370 \qquad \sum_{i=1}^{10} y_i = 116.5$$

$$\sum_{i=1}^{10} x_i^2 = 15888 \qquad \sum_{i=1}^{10} y_i^2 = 1532.65 \qquad \sum_{i=1}^{10} x_i y_i = 3834.7$$

(a) Using these summaries, find the sample correlation coefficient, and comment.
(b) Perform a linear regression analysis of *y* on *x*, and comment on the estimated slope term.
(c) This particular branch of Morrisons is in the process of taking on new checkout staff. Applicants for the new jobs have to undertake a series of aptitude tests, one of which requires them to be able to process at least 9 transactions per minute. Can we expect fifty year old applicant Joe King to pass this aptitude test?

Question 2

A random sample of five paired observations on annual returns for two mutual funds, labelled *X* and *Y* here, give the following summaries:

$$\bar{x} = 5.38 \qquad \bar{y} = 5.06$$

$$\sum_{i=1}^{5} x_i^2 = 572.31 \qquad \sum_{i=1}^{5} y_i^2 = 260.07 \qquad \sum_{i=1}^{5} x_i y_i = 354.26$$

(a) The correlation coefficient can be shown to be 0.918, and Minitab accompanies this with a *p*-value of 0.0278. Using this information, comment on the relationship between *X* and *Y*.
(b) Obtain the estimated regression equation from a simple linear regression of *Y* on *X*, and use this to estimate the annual return on *Y* given a return on *X* of –2.5.