**NEWCASTLE UNIVERSITY**

**SCHOOL OF MATHEMATICS & STATISTICS**

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**SEMESTER 2 2012/2013**

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**ACC1012**

# Professional Skills for Accounting and Finance

**Time allowed: 2 hours**

*Candidates should attempt all questions. Marks for each question are indicated. However you are advised that marks indicate the relative weight of individual questions, they do not correspond directly to marks on the University scale.*

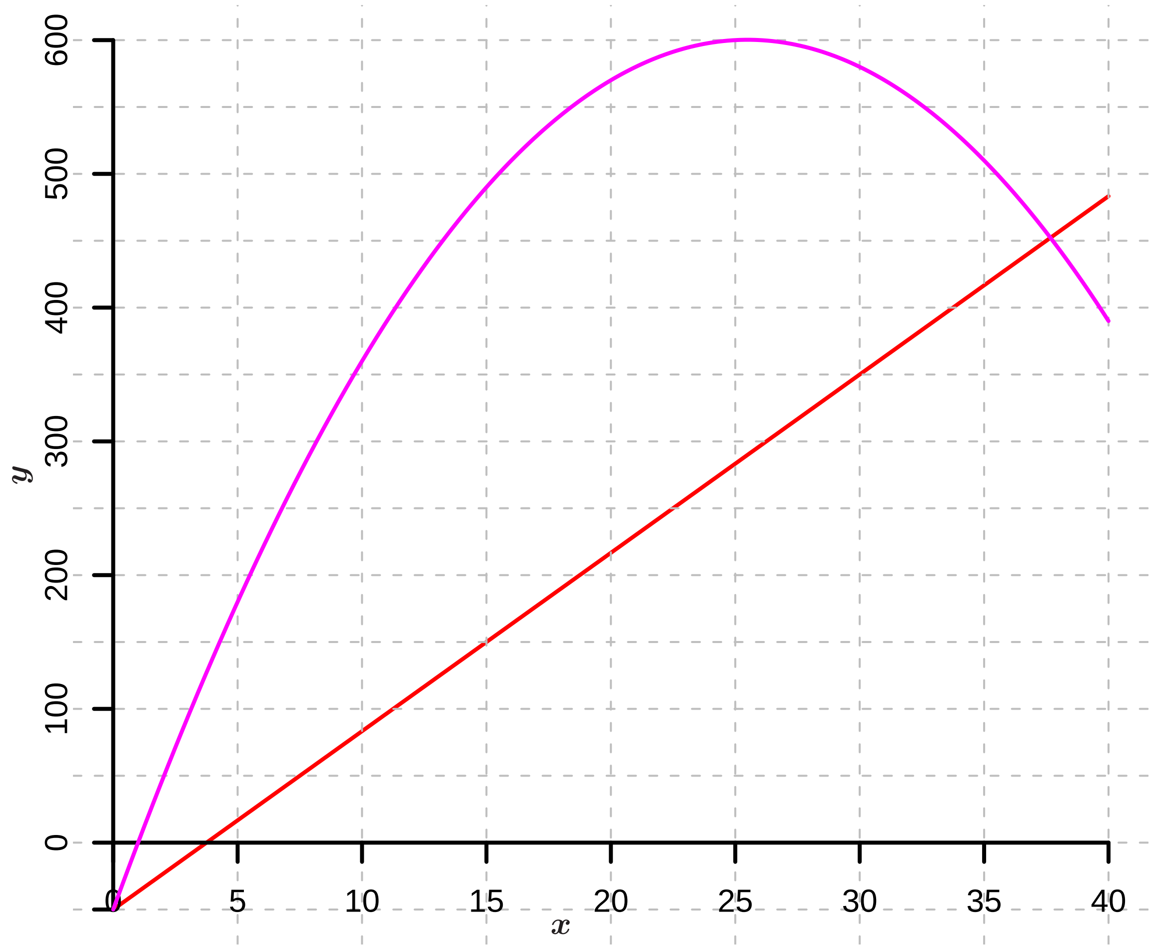
*There are SIX questions on this paper.*

*Answers to questions should be entered directly on this question paper in the spaces provided. This question paper must be handed in, attached inside an anonymised cover sheet, at the end of the examination.*

*A formula booklet, including statistical tables, will be provided.*

**1.** You work as a quantitative analyst for the *Financial Services Club* (FSC), an independent accounting firm for small businesses. In your current role, you use mathematical methods to forecast the profit margins of your clients.

(a) The graph below shows how your colleague believes total income (£$y$ thousand) will vary with advertising expenditure (£$x$ thousand) for one of FSC's clients in 2014.




Show that this linear income function can be written as $3y = 40x - 150$.

**[3 marks]**

**Answer:**

We clearly have a linear function. Suppose the general form of this linear function is $y = mx + c$. From the graph, we see that the $y$–intercept is $c = -50$. Also, the gradient $m$ is

$$m = \frac{\text{Change in } y}{\text{Change in } x} = \frac{350 - 150}{30 - 15} = \frac{40}{3},$$

giving

$$y = \frac{40}{3}x - 50;$$

multiplying throughout by 3 gives the required result.

*Question 1 continued on next page*

(b) Based on this linear income function, how much should this client spend on advertising in 2014 if they are to achieve a total income of at least £425,000?

[**2 marks**]

**Answer:**

We have

$$\frac{40}{3}x - 50 = 425.$$

Rearranging for $x$ gives

$$\begin{aligned} \frac{40}{3}x &= 475 \\ x &= \frac{3 \times 475}{40} = 35.625, \end{aligned}$$

i.e. at least £35,625.

Award only one mark if student gives a reading from the graph (won't be as accurate).

(c) After a consultation with this client, you believe a quadratic relationship between $x$ and $y$ might be more realistic. In particular, you specify that

$$y = -x^2 + 51x - 50.$$

Plot this quadaratic income function on the graph in part (a), for $0 \leq x \leq 40$. You may use the table of results given below to help you, if you wish.

[**3 marks**]

| $x$ | 10 | 20 | 25 | 30 | 40 |
|---|---|---|---|---|---|
| $y = -x^2 + 51x - 50$ | 360 | 570 | 600 | 580 | 390 |

*[This space has been left for any working you feel might be necessary]*

*Question 1 continued on next page*

(d) For this client you assume that, each year,

$$\text{Profit} = \text{Total income} - \text{Overheads}.$$

You also find that a suitable function for annual overheads, in terms of advertising expenditure $x$, is given by

$$\text{Overheads} = 19x + 90.$$

Using your quadratic function for total income in part (c), show that

$$P = -x^2 + 32x - 140,$$

where $P$ represents annual profit.

**[2 marks]**

**Answer:**

We have

$$\begin{aligned} \text{Profit} &= \text{Total income} - \text{Overheads} \\ &= -x^2 + 51x - 50 - (19x + 90) \\ &= -x^2 + 51x - 50 - 19x - 90 \\ &= -x^2 + 32x - 140, \end{aligned}$$

as required.

(e) How much should your client spend on advertising in 2014 to ensure they remain *in the black* (i.e. make a positive profit)?

**[4 marks]**

**Answer:**

We need to solve $P = -x^2 + 32x - 140 = 0$ for $x$. The discriminant $D$ is

$$D = 32^2 - (4 \times -1 \times -140) = 464,$$

giving

$$x = \frac{-32 \pm \sqrt{464}}{-2} = \frac{-32 \pm 21.541}{-2},$$

and so we have

$$x = 5.230 \quad \text{and} \quad x = 26.770$$

when $P = 0$. Thus, to make sure they remain *in the black*, our client should spend anywhere between £5,230 and £26,770 on advertising.

*Question 1 continued on next page*

(f) (i) Find $\frac{dP}{dx}$.

**[2 marks]**

**Answer:**

$$\frac{dP}{dx} = -2x + 32.$$

(ii) Hence, find your client's optimal advertising expenditure (i.e. the advertising expenditure which maximises profit).

**[3 marks]**

**Answer:**

To maximise, we set the gradient function equal to zero and solve for $x$, giving

$$\begin{aligned} -2x + 32 &= 0 \\ 2x &= 32 \\ x &= 16. \end{aligned}$$

Thus, our client should spend £16,000 on advertising to maximise profit.

To check this is a maximum:

$$\frac{d^2P}{dx^2} = -2,$$

which is negative for all values of $x$. Thus, we have a maximum.

(iii) What can your client expect their maximum profit to be in 2014, if they spend the optimal amount on advertising?

**[1 mark]**

**Answer:**

When $x = 16$ we have

$$P = -(16^2) + 32 \times 16 - 140 = 116,$$

that is, £116,000.

**[Total Q1: 20 marks]**

**2**. The *Monster Party Company* produce two types of party pack. Their "Ghastly" party pack contains 10 balloons and 64 sweets. Their "Devilish" party pack contains 20 balloons and 16 sweets. Each day, the company has 3000 balloons and 8000 sweets available.

Every day, the company sells at least 50 of each type of party pack; in total, they make and sell $x$ "Ghastly" party packs and $y$ "Devilish" party packs. The company sells each "Ghastly" party pack at a profit of £1.20 and each "Devilish" party pack at a profit of £1.80.

(a) Formulate the *Monster Party Company*'s situation as a linear programming problem.

**[6 marks]**

**Answer:**

We have already been told that

$$\begin{aligned} x &= \text{No. of "Ghastly" party packs} \\ y &= \text{No. of "Devilish" party packs} \end{aligned}$$

Also,

| | Balloons | Sweets | Profit (pence) |
|---|---|---|---|
| "Ghastly" | 10 | 64 | 120 |
| "Devilish" | 20 | 16 | 180 |
| Limits | 3000 | 8000 | |

This gives the following **constraints**:

$$\begin{aligned} 10x + 20y &\leq 3000 \\ 64x + 16y &\leq 8000 \end{aligned}$$

Also, we are told that

$$\begin{aligned} x &\geq 50 \\ y &\geq 50 \end{aligned}$$

The **objective function** is:

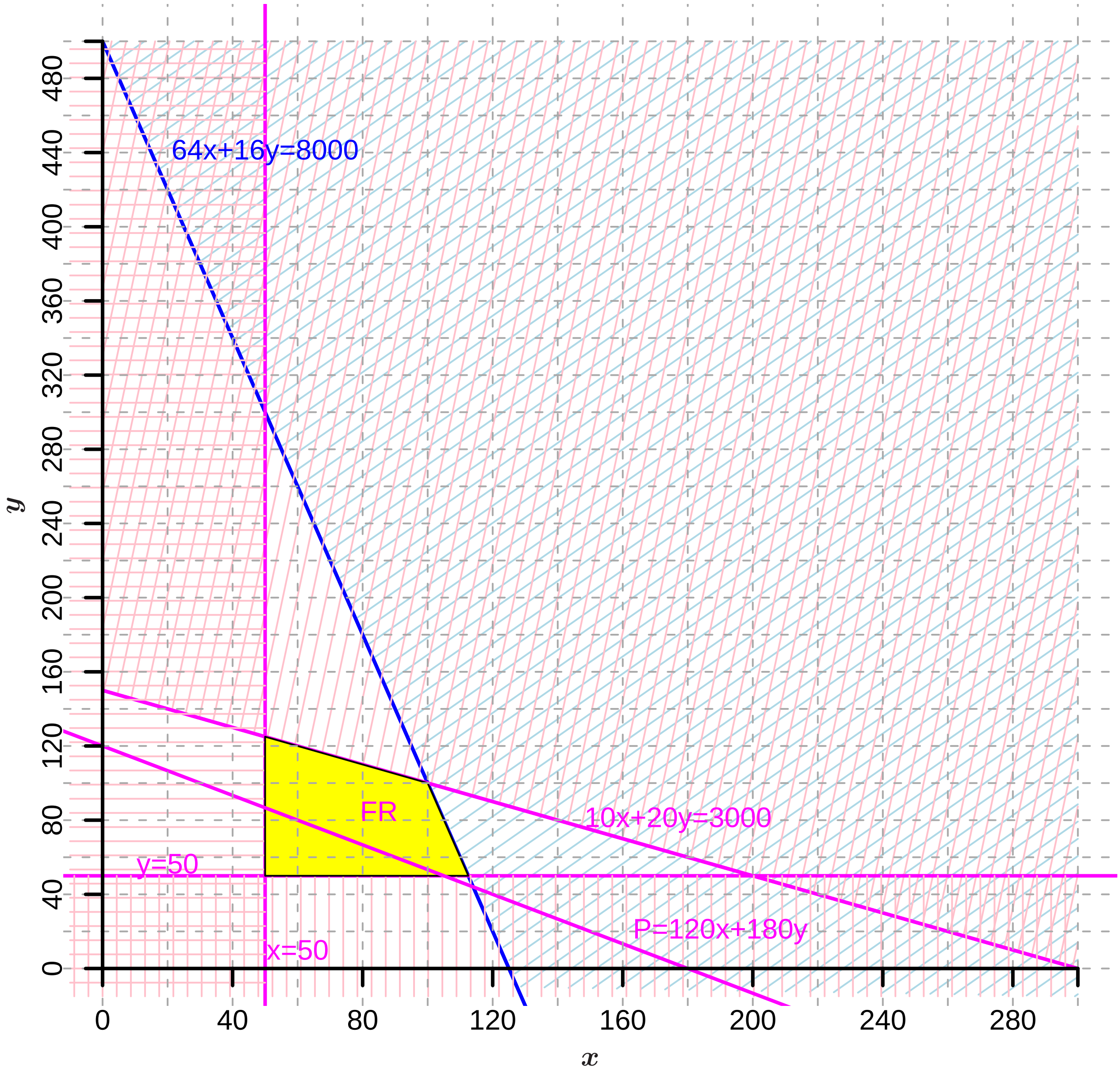
$$P = 120x + 180y,$$

where $P$ represents daily profit.

*Question 2 continued on next page*

(b) Complete the graph below to represent the linear programming problem you formulated in part (a). Make sure you clearly indicate the feasible region, as well as the direction of the objective function. *[You may use the space underneath to show any working you feel might be necessary]*

**[7 marks]**



(c) Using your graph in part (b), find the company's maximum daily profit, £$P$.

**[2 marks]**

**Answer:**

From the graph, we have

$$\text{Maximum profit} \ : \ (x = 100, y = 100) \longrightarrow P = 100 \times 120 + 100 \times 180 = £300$$

(d) Now solve this problem algebraically to verify the company's maximum daily profit obtained in part (c).

**[3 marks]**

**Answer:**

We solve, simultaneously,

$$\begin{aligned} 10x + 20y &= 3000 \qquad (1) \\ 64x + 16y &= 8000 \qquad (2) \end{aligned}$$

Multiplying (1) by 4 and (2) by 5, we get:

$$\begin{aligned} 40x + 80y &= 12000 \qquad (3) \\ 320x + 80y &= 40000 \qquad (4) \end{aligned}$$

Subtracting (3) from (4) gives

$$280x = 28000 \longrightarrow x = 100.$$

Substituting $x = 100$ into (1) gives

$$\begin{aligned} 1000 + 20y &= 3000 \\ 20y &= 2000 \\ y &= 100, \end{aligned}$$

as required.

**[Total Q2: 18 marks]**

**3**. In October 2010 model and celebrity Kate Moss endorsed the latest *Top Shop* women's clothing range, where she starred in an advertising campaign aimed at increasing sales (see pictures below – © *TopShop* 2010). Total sales of women's clothing at a selection of *Top Shop* stores are summarised in the table underneath – for periods *before* and *after* this advertising campaign.



| **Total sales** | **Percentage** | | **Cumulative percentage** | |
|---|---|---|---|---|
| ($X$ thousand pounds) | Before | After | Before | After |
| $10 \leq x < 14$ | 12 | 2 | 12 | 2 |
| $14 \leq x < 18$ | 32 | 15 | 44 | 17 |
| $18 \leq x < 22$ | 40 | 22 | 84 | 39 |
| $22 \leq x < 26$ | 10 | 38 | 94 | 77 |
| $26 \leq x < 30$ | 5 | 17 | 99 | 94 |
| $30 \leq x < 34$ | 1 | 4 | 100 | 98 |
| $34 \leq x < 38$ | 0 | 2 | 100 | 100 |

(a) Write down the modal class for sales in each of the before and after periods.

**[1 mark]**

**Answer:**

Modal class before : $£18,000 \longrightarrow £22,000$

Modal class after : $£22,000 \longrightarrow £26,000$

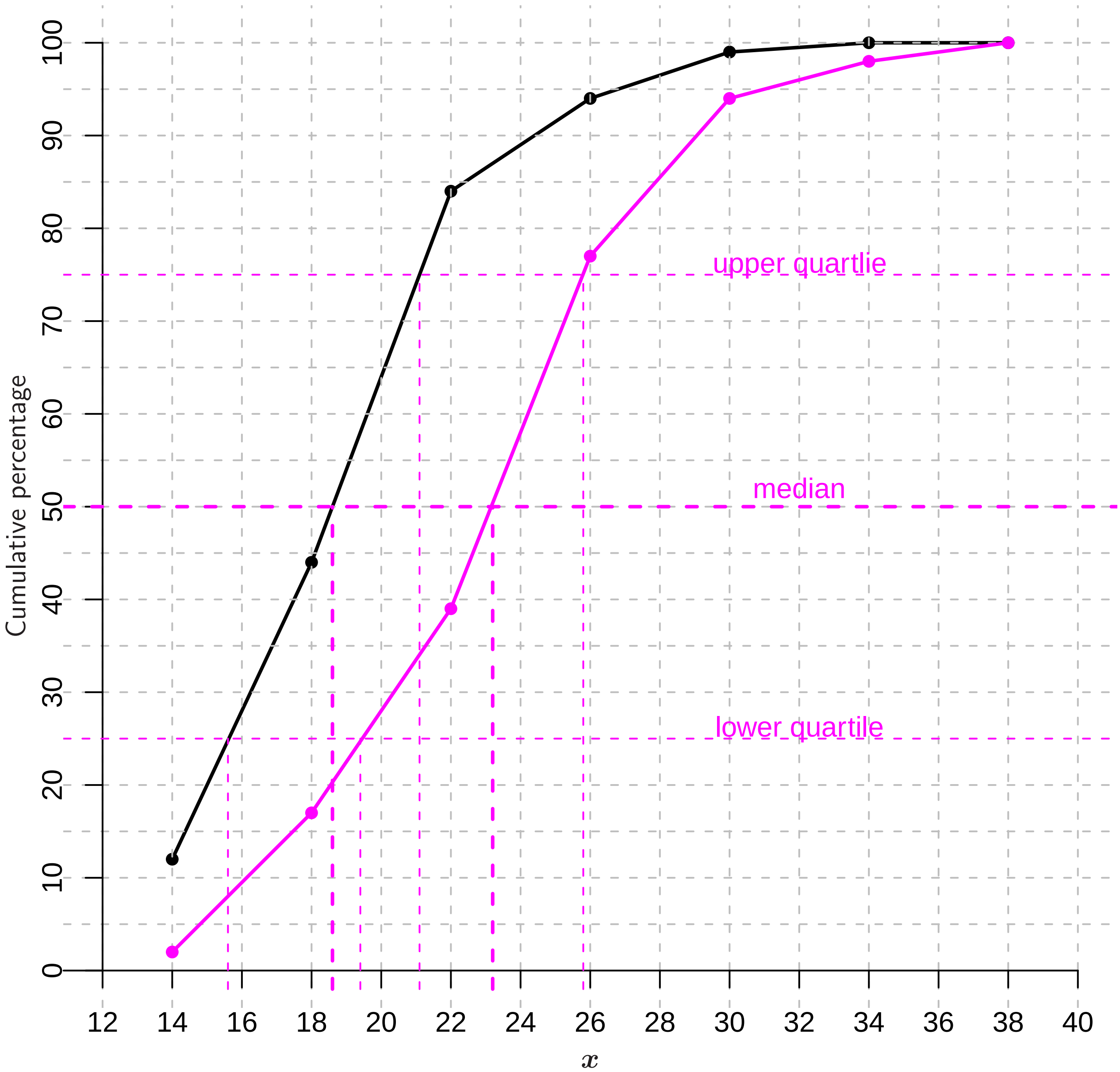
(b) Complete the table above by calculating the cumulative percentages for sales in the period after the advertising campaign.

**[3 marks]**

*Question 3 continued on next page*

(c) The graph below shows the ogive (cumulative relative frequency polygon) for sales in the before period. On the same graph, construct the ogive for sales in the after period.

**[3 marks]**



(d) Using the ogives from part (c),

(i) estimate the median level of sales for both the before and after periods;

**[2 marks]**

**Answer:**

$$\text{Median (before)} = \pounds 18,600$$

$$\text{Median (after)} = \pounds 23,200$$

(ii) estimate the inter–quartile range for sales in the before and after periods.

**[4 marks]**

**Answer:**

Using the graph, we can estimate the lower and upper quartiles for "before" and "after", giving:

$$IQR \text{ (before)} = 21.1 - 15.6 = \pounds 5,500, \qquad \text{and}$$

$$IQR \text{ (after)} = 25.8 - 19.4 = \pounds 6,400.$$

(e) Using the ogives in part (c) and the summaries in parts (a) and (d), compare and contrast sales at *Top Shop* before and after the advertising campaign.

**[3 marks]**

**Answer:**

The advertising campaign has obviously had a positive effect on sales – the ogive for the "after" period is wholly to the right of the that for the "before" period. This is can also be seen in the measures of average obtained in parts (a) and (d) – median sales in the after period are more than £4,500 higher than in the before period, and the modal class is also higher for the after period. However, sales in the after period at this selection of stores are more dispersed.

*Question 3 continued on next page*

(f) Use the percentages given in the table to show that mean sales in the after period can be approximated as £24,900. (to the nearest hundred pound). Why is your answer an *approximation*?

**[3 marks]**

**Answer:**

$$\bar{x} \approx \frac{(2 \times 14) + (15 \times 18) + \ldots + (2 \times 38)}{100} = 24.92,$$

i.e. $£24,920 \approx 24,900$ , as required. This is an approximation because we are assuming that all values in each class interval take the mid–point, which of course will probably not be the case.

(g) It can be shown that the standard deviation for sales in the after period can be approximated as £4,800. Assuming that

$$X \sim N(24.9, 4.8^2),$$

find the probability that, at a randomly chosen *Top Shop* store, sales exceed £35,000.

**[3 marks]**

**Answer:**

$$\begin{aligned} P(X > 35) &= 1 - P(X \leq 35) \\ &= 1 - P\left(Z \leq \frac{35 - 24.9}{4.8}\right) \\ &= 1 - P(Z \leq 2.10) \\ &= 1 - 0.9821 \\ &= 0.0179. \end{aligned}$$

**[Total Q3: 22 marks]**

4. You work as an IT analyst for the *Lonely Globe* online travel company. One of *Lonely Globe*'s travel writers has recently posted a travel video to the company's webpage showcasing "The best of New York City in 2013". Suppose you are interested in $Y$, the number of downloads of this video, in the first hour of it being posted.

   (a) Would the binomial distribution or the Poisson distribution be the most appropriate probability model for $Y$? Tick the correct box, and briefly explain your choice.
   **[3 marks]**

   **Answer:**

   Binomial ☐ Poisson ☐

   Brief explanation: We are interested in the number of downloads over a fixed time interval, and there is no (known) upper limit. We do not have a fixed number of trials, with known success probability, as required for the binomial distribution.

   (b) A similar video featuring New Orleans, uploaded last month, had 5 downloads in the first hour of it being posted. Assuming that downloads for the New York video will occur at the same rate, find the probability that there will be fewer than two downloads in the first hour.
   **[5 marks]**

   **Answer:**

   We are assuming $Y \sim \text{Po}(5)$. Thus

$$\begin{aligned} P(Y < 2) &= P(Y = 0) + P(Y = 1) \\ &= \frac{e^{-5}5^0}{0!} + \frac{e^{-5}5^1}{1!} \\ &= 0.0067 + 0.0337 \\ &= 0.0404. \end{aligned}$$

   (c) Suppose you are now interested in the number of hourly downloads of the New York video a week after it has been posted. Why might the model you used in part (b) no longer be appropriate?
   **[2 marks]**

   **Answer:**

   We might expect the hourly download rate of this video to change a week after it was initially made available.

**[Total Q4: 10 marks]**

**5**. Before launching a new product, small businesses usually conduct market research to assess the viability of this new product. As a business analyst, you are interested in whether or not there is a difference in the amount spent on such market research by small companies in Northeast and Southeast England. From a random sample of small businesses in both regions, the table below summarises the amount spent (in thousands of pounds):

| Region | Sample size | Mean | Standard deviation |
|---|---|---|---|
| Northeast | 15 | 86.4 | 5.8 |
| Southeast | 12 | 90.5 | 4.5 |

(a) The 95% confidence interval for the population mean expenditure in the Northeast is $(83.19, 89.61)$ thousand pounds. Calculate the equivalent confidence interval for the Southeast.

**[5 marks]**

**Answer:**

For the Southeast, we have

$$\begin{aligned} 90.5 &\pm \mathbf{2.201}\sqrt{\frac{4.5^2}{12}} \quad \text{i.e.} \\ 90.5 &\pm 2.859, \end{aligned}$$

giving $(87.64, 93.36)$ thousand pounds.

(b) Perform an appropriate hypothesis test to determine whether there is a difference in the average amount small businesses spend on market research between the regions. *[Hint: the pooled standard deviation is $s = 5.268$]*

**[10 marks]**

**Answer:**

We test

$$\begin{aligned} H_0 &: \mu_{NE} = \mu_{SE} \quad \text{versus} \\ H_1 &: \mu_{NE} \neq \mu_{SE}. \end{aligned}$$

The test statistic is

$$t = \frac{|86.5 - 90.4|}{5.268 \times \sqrt{\frac{1}{15} + \frac{1}{12}}} = 1.914.$$

*Question 5 continued on next page*

*[Part of this page has been left blank for your solution to the last question]*

From $t$–tables on $\nu = 25$ degrees of freedom, we get:

| $p$–value | 10% | 5% | 1% |
|---|---|---|---|
| Critical value | 1.708 | 2.060 | 2.787 |

Thus, $5\% < p < 10\%$. So

- There is only *slight* evidence against $H_0$
- We therefore *retain* $H_0$
- There is insufficient evidence to suggest a real difference in the amount spent on market research in the Northeast and Southeast regions.

(c) Briefly explain why the confidence intervals from part (a) support your findings in part (b).

**[3 marks]**

**Answer:**

The confidence intervals for the Northeast and Southeast overlap – suggesting any differences observed in the samples are probably just due to chance fluctuations rather than real differences in the populations. This is exactly our conclusion in the hypothesis test!

**[Total Q5: 18 marks]**

**6**. *VIX* is a trademarked stock symbol for the *Chicago Board Options Exchange Market Volatility Index*, a popular measure of the implied volatility of S&P 500 index options. For one company, daily *VIX* indices ($x$) were compared to the corresponding S&P 500 one–day returns ($y$) for that company, over a period of 14 days. The following summaries were obtained:

$$\sum_{i=1}^{14} x_i = 408.97 \qquad \sum_{i=1}^{14} y_i = 43.90$$

$$\sum_{i=1}^{14} x_i^2 = 13612.27 \qquad \sum_{i=1}^{14} y_i^2 = 166.21 \qquad \sum_{i=1}^{14} x_i y_i = 1443.55$$

(a) Perform a linear regression of $y$ on $x$, clearly stating your regression equation in the form

$$y = \beta_0 + \beta_1 x + \epsilon,$$

where $\epsilon \sim N(0, \sigma^2)$.

**[10 marks]**

**Answer:**

We have

$$\begin{aligned} S_{xy} &= \sum_{i=1}^{14} x_i y_i - n\bar{x}\bar{y} \\ &= 1443.55 - 14 \times \frac{408.97}{14} \times \frac{43.90}{14} = 161.137 \\ S_{yy} &= \sum_{i=1}^{14} y_i^2 - n\bar{y}^2 \\ &= 166.21 - 14 \times \left(\frac{43.90}{14}\right)^2 = 28.552 \\ S_{xx} &= \sum_{i=1}^{14} x_i^2 - n\bar{x}^2 \\ &= 13612.27 - 14 \times \left(\frac{408.97}{14}\right)^2 = 1665.380. \end{aligned}$$

This gives

$$\hat{\beta}_1 = \frac{S_{xy}}{S_{xx}} = \frac{161.137}{1655.380} = 0.097,$$

and

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1\bar{x} = \frac{43.90}{14} - 0.097 \times \frac{408.97}{14} = 0.302,$$

giving

$$y = 0.302 + 0.097x + \epsilon.$$

*Question 6 continued on next page*

(b) Forecast the S&P 500 one day return for a company with a *VIX* of 30.5.
[**2 marks**]

**Answer:**

$$y = 0.302 + 0.097 \times 30.5 = 3.261.$$

[**Total Q6: 12 marks**]

**THE END**