ACC1012/ACC1013:
Professional Skills for Accounting and Finance

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Refresher Mathematics & Statistics material for new
Accounting & Finance/Business Accounting & Finance students

Newcastle University Business School
Newcastle University, Newcastle upon Tyne, U.K.

September 2013 entry

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Congratulations to those who have just got a place at Newcastle University! In September, as a new Accounting & Finance/Business Accounting & Finance student, you will start a module called:

ACC1012/ACC1013: Professional Skills for Accounting and Finance

One of the main aims of this module is to equip you with the basic skills in Mathematics and Statistics that you will need throughout the rest of your degree course. The main aim of this revision booklet is to let you know what we'll expect of you, in terms of Maths and Stats, before you come to Newcastle in September! Before you join us in September, we'd like you to spend some time reading through this revision booklet – please try out the questions! Throughout the notes, there are two types of questions:

Worked example – please read!

The solution will be shown in red.

These questions are for you to try – no solution will be given!

Fill in the gaps where you see the pencil icon!

✎

You might find material in the sections marked with an asterisk (*) more challenging – if so, don't worry! Support with these topics will be provided at the start of the module, just to get everyone up–to–speed!

At the end of each section, there will be a list of practice questions for you to try – full solutions are provided at the end of the booklet. For more practice, once you've worked through this booklet we recommend that you try our online Numbas test – see the webpage for more details.

Contents

Chapter 1

Basic maths skills

1.1 A reminder of some useful skills

In this section you will be reminded of some of the basic (pre–GCSE) maths skills that will come in handy for the Professional Skills module – and indeed *any* course you will take at University which contains elements of numeracy. It all might seem pretty basic – but these topics have been included because we know it's often the simple stuff that trips students up!

1.1.1 BIDMAS

When performing mathematical calculations it is important to remember the order in which you perform certain operations. "BIDMAS" is a common mnemonic used in schools to help students remember the order in which to perform mathematical operations – namely: **B**rackets, **I**ndices, **D**ivision, **M**ultiplication, **A**ddition, **S**ubtraction. "Indices" is another word for "powers". Remember, 2^3 means "2 to the power of 3", sometimes referred to as "2 cubed", and is worked out as

$$2^3 = 2 \times 2 \times 2 = 8.$$

Consider the expression

$$y = 3 \times 6 + 4.$$

Observing the rules of BIDMAS we get $y = 18 + 4 = 22$. Notice we have done the multiplication first! If we'd done the addition first, we would have:

$$\begin{aligned} y &= 3 \times 6 + 4 \\ &= 3 \times 10 \\ &= 30, \end{aligned}$$

which is incorrect! However, if we had brackets around the $(6 + 4)$, then

$$\begin{aligned} y &= 3 \times (6 + 4) \\ &= 3 \times 10 \\ &= 30. \end{aligned}$$

What about

$$(5 + 6 \div 2) \times 3^2?$$

BIDMAS tells us to do the brackets first, giving

$$5 + 6 \div 2 = 5 + 3 = 8,$$

as we need to do division before addition. So now we have

$$8 \times 3^2.$$

Next, according to BIDMAS, would be the power. $3^2 = 3 \times 3 = 9$, and so we now have

$$8 \times 9 = 72 \text{ as our final answer.}$$

1.1.2 Negative numbers

Just be careful with negatives! Some simple examples:

1. $7 - 15 = -8$

2. $-10 - 4 = -14$

3. $-6 + 8 = 2$

4. $8 - (-4) = 8 + 4 = 12$

5. $3 + (-10) = 3 - 10 = -7$

6. $4 \times (-3) = -12$

7. $-20 \div -2 = 10$

Just remember: multiplying/dividing two negatives gives a positive; multiplying/dividing a positive by a negative, or a negative by a positive, gives a negative!

1.1.3 Fractions

For the Professional Skills course it will be very useful if you can remember how to manipulate fractions. You've all done this sort of thing before, but you may have forgotten!

Multiplication/division of fractions

What is

$$\frac{2}{3} \times \frac{3}{8}?$$

To multiply together two fractions, simply multiply the two numbers on the top (the *numerators*) and multiply the two numbers on the bottom (the *denominators*), giving

$$\frac{2}{3} \times \frac{3}{8} = \frac{2 \times 3}{3 \times 8} = \frac{6}{24}.$$

Now the answer – $\frac{6}{24}$ – is not in it's lowest terms. Both 6 and 24 are multiples of 6, and so we can simplify the answer by dividing both top and bottom by 6:

$$\frac{6}{24} = \frac{\cancel{6}^1}{\cancel{24}^4} = \frac{1}{4},$$

or a quarter.

What about division? Suppose we want to work out

$$\frac{1}{4} \div \frac{4}{9}.$$

The trick is to "invert" the second fraction, and then multiply, giving:

$$\frac{1}{4} \times \frac{9}{4} = \frac{1 \times 9}{4 \times 4} = \frac{9}{16},$$

which cannot be simplified any further.

Addition/subtraction of fractions

Suppose we want to work out

$$\frac{3}{5}+\frac{1}{5}.$$

Since both fractions have the same denominator, it's really easy – just add together the two numerators and keep the denominator the same, that is:

$$\frac{3}{5}+\frac{1}{5}=\frac{3+1}{5}=\frac{4}{5}.$$

What about

$$\frac{11}{16}+\frac{7}{12}?$$

Now the denominators are different, so we have to make them the same! After a bit of thought, you should see that both numerators are factors of 96 – that is, both 16 an 12 "go into" 96. So we will initially make both fractions into fractions with a denominator of 96. To do this, we need to multiply both top and bottom of the first fraction by 6, giving:

$$\frac{11}{16}\times\frac{6}{6}=\frac{66}{96}.$$

For the second fraction, we need to multiply both top and bottom by 8:

$$\frac{7}{12}\times\frac{8}{8}=\frac{56}{96}.$$

Notice that multiplying by $\frac{6}{6}$ or $\frac{8}{8}$ is the same as multiplying by 1, and multiplying anything by 1 just keeps it the same. Now we can add:

$$\frac{66}{96}+\frac{56}{96}=\frac{122}{96}.$$

Now we can simplify. Both numerator and denominator are multiples of 2:

$$\frac{\not{122}^{61}}{\not{96}^{48}}=\frac{61}{48},$$

which cannot be simplified any further. However, it *is* "top heavy", so we might break it down as

$$\frac{61}{48}=\frac{48}{48}+\frac{13}{48}=1\frac{13}{48}.$$

To subtract two fractions, you would follow exactly the same procedure! There are lots more examples on fractions in the Numbas test. If you have more than two fractions and more than two operators, e.g. a mixture of $+, -, \times, \div$, just bear the rules of BIDMAS in mind!

1.1.4 Percentages

You should be able to work out simple percentages and percentage changes. For example, suppose you need to find 21% of £250. We can write 21% as a fraction out of 100, and then multiply it by £250:

$$\frac{21}{100}\times 250=\frac{21\times 250}{100}=52.5,$$

or £52.50.

Suppose annual profits at a company were £50,000 and £61,000 in 2012 and 2013 (respectively). The percentage change is given by

$$\frac{61,000-50,000}{50,000}\times 100\%=\frac{11,000}{50,000}\times 100\%=22\%.$$

1.1.5 Basic algebra

Often in maths we use letters as a shorthand representation for something else – for example, x could represent profit, or income, or costs. You will need to be comfortable with basic manipulation of algebraic expressions. For example,

1. $2x + 5x = 7x$

2. $4y - 10y = -6y$

3. $3s + 5t - 5s + 2t = -2s + 7t$

4. $4t + 6 \times 2t = 4t + 12t = 16t$

Just bear in mind the rules of BIDMAS and negative numbers! You should also be able to work out an expression numerically on substitution of a letter with a number. For example, if $x = 2$, $y = 3$, $s = 10$ and $t = 100$, then after substitution we would get $7 \times 2 = 14$ in question 1, $-6 \times 3 = -18$ in question 2, $-2 \times 10 + 7 \times 100 = -20 + 700 = 680$ in question 3 and $16 \times 100 = 1,600$ in question 4.

1.1.6 Expansion of brackets

In mathematics it is common to see expressions which include brackets – normally used as a shorthand way of writing longer statements. The brackets usually mean that the formula has been *factorised* in some way – that is, common factors have been pulled from each element in the equation (see the next section). For example, we may see an expression like

$$2(x + 2).$$

All that this means is that we need to multiply each of the arguments within the brackets by the argument outside:

$$2(x + 2) = 2 \times x + 2 \times 2 = 2x + 4.$$

We may also have something like the following:

$$(x + 1)(x + 2).$$

Here, we apply the same idea: times everything inside the first pair of brackets by everything inside the second pair of brackets, and then add them up. Here it may be useful to remember another acronym: "FOIL". This stands for **F**irst, **O**utside, **I**nside, **L**ast. If we take the expression above as an example:

$$\begin{aligned} (x + 1)(x + 2) = & \\ &= \underbrace{x \times x}_{\text{first two}} + \underbrace{x \times 2}_{\text{outside two}} + \underbrace{1 \times x}_{\text{inside two}} + \underbrace{1 \times 2}_{\text{last two}} \\ &= x^2 + 2x + x + 2 \\ &= x^2 + 3x + 2 \end{aligned}$$

Notice that the highest power of x is a 2, and so we have a *quadratic*; we will come back to quadratics later on.

1.1.7 Factorisation

Factorisation is the notion of taking out common *factors* from a mathematical expression. A factor is just something which divides something else exactly (e.g. 3 is a factor of 12). Factorisation can be useful to help see how equations might be simplified. For example, consider:

$$6y = 3x + 15.$$

We could factorise the right hand side of this equation as

$$\begin{aligned} 6y &= 3 \times x + 3 \times 5 \\ &= 3(x + 5). \end{aligned}$$

Doing this then allows us to simplify the expression to

$$\begin{aligned} 3 \times 2y &= 3(x + 5) \\ \cancel{3} \times 2y &= \cancel{3}(x + 5) \\ 2y &= x + 5. \end{aligned}$$

1.2 Linear functions

1.2.1 Introduction

Linear functions occur frequently in accounting and finance. You should already have at least some familiarity with linear functions from your GCSE maths course, or equivalent, at school. For example,

$$y = 3x + 4$$

is a linear function of x which, when operated, gives y. When $x = 6$ say, we can apply the linear function to get

$$\begin{aligned} y &= 3 \times 6 + 4 \\ &= 22. \end{aligned}$$

Suppose $y = 2(5x + 3) + (2x - 1)$. What is the value of y when $x = 4$?

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For the function in the question above it is possible to simplify the expression algebraically. Although this will not be a main focus of the module, it can be a useful skill. You may remember from your courses at school, and your work in Section 1.1 of this booklet, that we could do the following:

$$\begin{aligned} y &= 2(5x + 3) + (2x - 1) \\ &= 10x + 6 + 2x - 1 \\ &= 12x + 5. \end{aligned}$$

Just like the function given at the start ($y = 3x + 4$), this too is a *linear* function. This is because if we plot the function on a graph (see the next section) we get a straight line. Simple linear functions have an *intercept* and a *gradient*. The intercept is the point at which the line crosses the y-axis on the graph; the gradient tells us about the slope of the graph – specifically, how steep it is and in which direction it is travelling. More generally, the linear function

$$y = mx + c$$

has intercept c and gradient m.

What are the intercept and gradient of the two linear functions that have been discussed so far? Which do you think will give you the "steepest" line?

Notice that the gradient is always the number next to the x; namely, the gradient is the *coefficient* of x, or x coefficient. The intercept is the "other" number, or is often referred to as the *constant*. As we shall see shortly, if the x coefficient is positive, $(m > 0)$, the graph has an "uphill" slope. Conversely if it's negative, $(m < 0)$, it has a "downhill" slope. A larger coefficient leads to a steeper slope.

What are the gradient and intercept of the following linear functions?

1. $y = 7x - 2$
2. $y = 5 + 3x$
3. $y = 2(3 + x) - 5x$
4. $2y = 8x + 4$

1.2.2 Graphs of linear functions

Let's now consider how to draw the graph of a linear function. Consider the first linear function that we looked at, $y = 3x + 4$. One way to draw a graph of this function is to consider the function at various values of x and, by substitution, work out the corresponding y values. We can then plot x against y and join up the points. For example, in table 1.1 we have started a sequence of x values, and then worked out the corresponding y values by finding $3x + 4$ for each x value in our sequence.

x	−1	0	1	2	3	4	5	6	7
$y = 3x + 4$	1	4	7	10	13	16	19	22	25

Table 1.1: Table of values corresponding to $y = 3x + 4$

Plotting x against y and joining up the points gives the graph shown below. Notice that the graph is linear, i.e. we get a straight line, so we could have produced the same graph using only two points instead of the 9 above (it's true that you can draw any straight line in x and y using only two points).

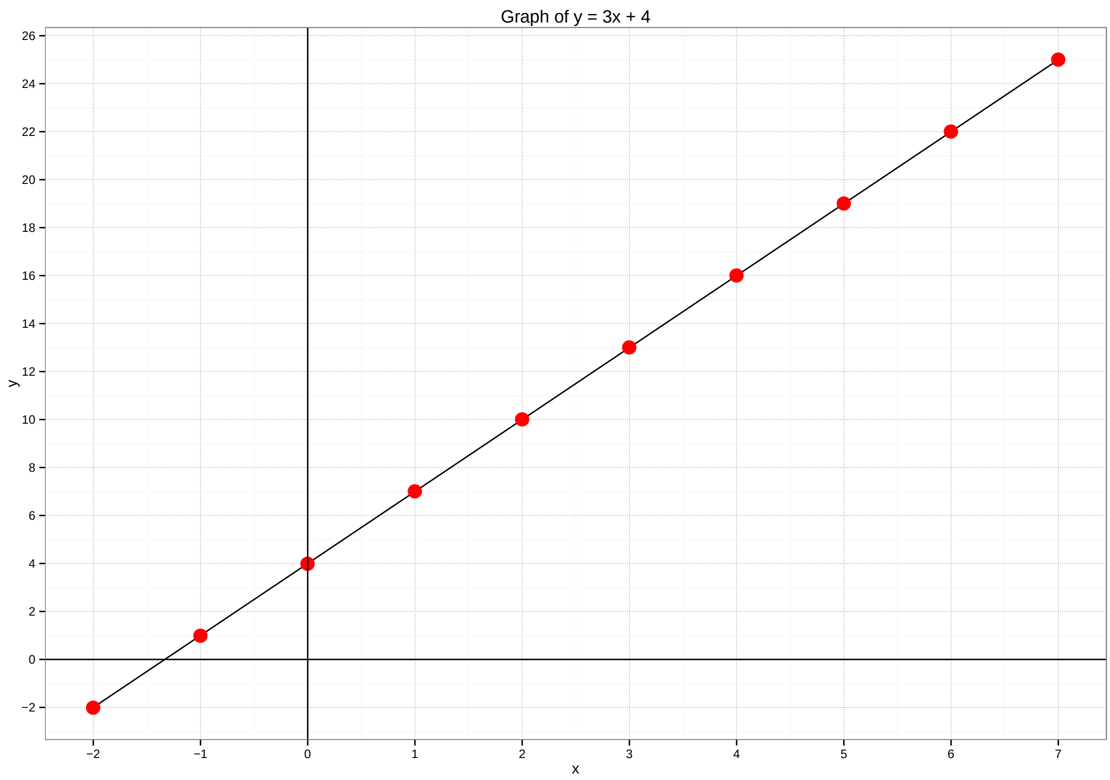

Figure 1.1: Graph of the linear function $y = 3x + 4$.

In the space below, draw the graph of the function $y = -2x + 10$. Remember that it is possible to do this using only 2 points, and you should already know the y-intercept.

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A company believes it's net monthly income (y pounds) is related to how much it spends on advertising (x pounds).

In fact, in any particular month the company's gross income will be five times the amount it spends on advertising, although monthly overheads of £15,000 should be deducted to obtain total income after costs (i.e. the net income).

(a) Write down the linear equation linking net income to advertising expenditure and overheads.

(b) What are the intercept and gradient?

(c) Plot a graph of your net income.

(d) Using the graph, or the equation, what can we expect net income to be this month if we spend £15,000 on advertising?

(a) We know that the company's gross income will be five times the amount spent on advertising (x pounds), so we can write this as $5x$. However we want net monthly income – gross income minus initial costs. Knowing that the initial costs are £15,000, we get the equation

$$y = 5x - 15000$$

(b) We know from this revision booklet that finding the intercept and gradient are easy. The intercept is the constant term and the gradient is the x coefficient. Hence

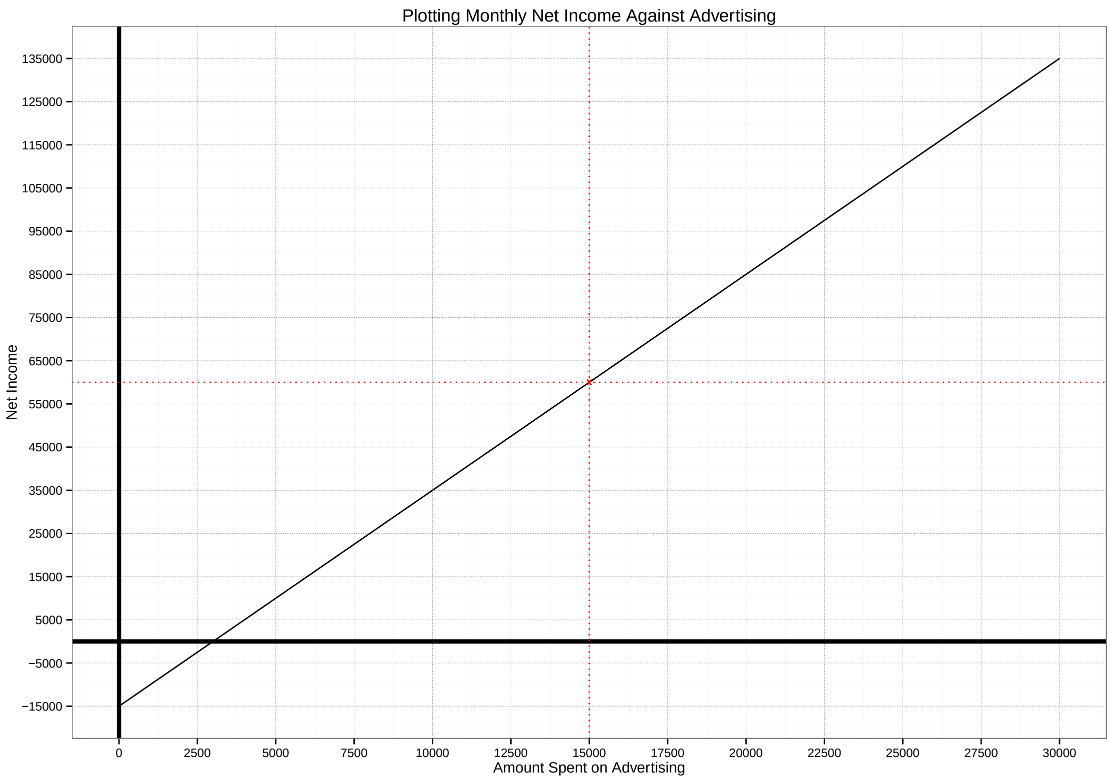
$$\text{Intercept} = -15000$$
$$\text{Gradient} = 5$$

(c) We saw earlier that we could plot any straight line by considering some values of x and finding the corresponding y values. We also saw that we could do this using only two points. Consider $x = 0$ and $x = 30000$:

If $x = 0$	If $x = 30000$
$y = 5 \times 0 - 15000$	$y = 5 \times 30000 - 15000$
$y = -15000$	$y = 135000.$

So we will have a straight line connecting these two points.

(d) We can substitute $x = 15000$ into the equation giving $y = 5 \times 15000 - 15000 = 60000$; so we can expect a net income of £60,000. We could also have done this by using the graph: go along to 15000 on the x-axis, follow it up to the line, when we reach the line go straight across to the y-axis to find our value of y.



1.2.3 Finding the equation of a line

You should now feel comfortable with the general equation of a line, that is,

$$y = mx + c,$$

where c is the y-intercept and m is the gradient. You should also be able to plot such a line on a graph. In this section we will also consider how to go the other way – given a plot of the line, what's the equation? Consider the line in the plot on the left in figure 1.2. Suppose we need to work out the equation of the line. In order to write this down we need the y-intercept and the gradient. Clearly the line intersects the y-axis at 5, and so we know that in our equation of the line $c = 5$. But what about the gradient? You may be familiar with the following gradient formula from your GCSE math course:

$$\text{Gradient} = \frac{\text{Change in } y}{\text{Change in } x}.$$

In order to work out the "Change in x" and the "Change in y" we need to consider the x and y co–ordinates at two points on the graph. For example, we can see that the line passes through $(4, 7)$ and $(10, 10)$. The values in brackets represent the co–ordinates of the point where the first value is the x co–ordinate and the second value is the y co–ordinate. Thus

$$m = \frac{10 - 7}{10 - 4} = \frac{3}{6} = \frac{1}{2} = 0.5,$$

giving us the linear equation $y = \frac{1}{2}x + 5$.

Now look at the line on the right in figure 1.2.

1. Is the gradient positive or negative?
2. Is the line more steep or less steep than the line on the left?
3. Find the equation of this line.

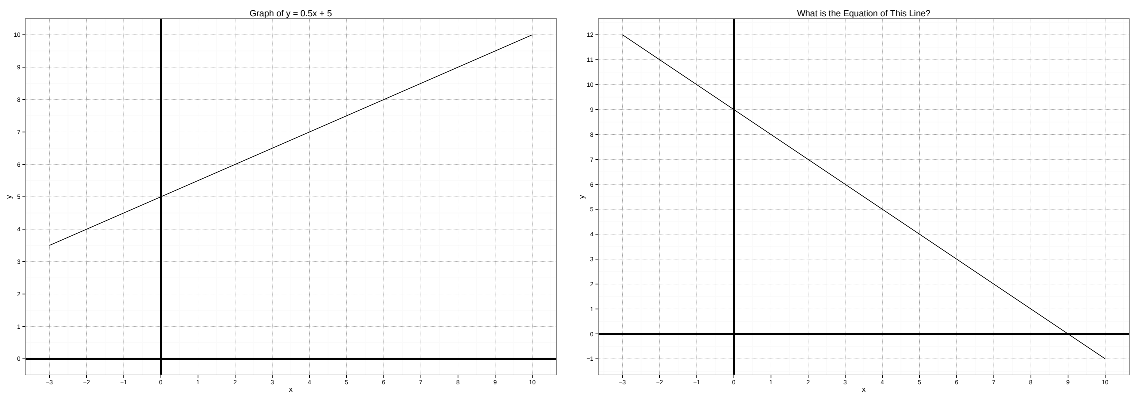
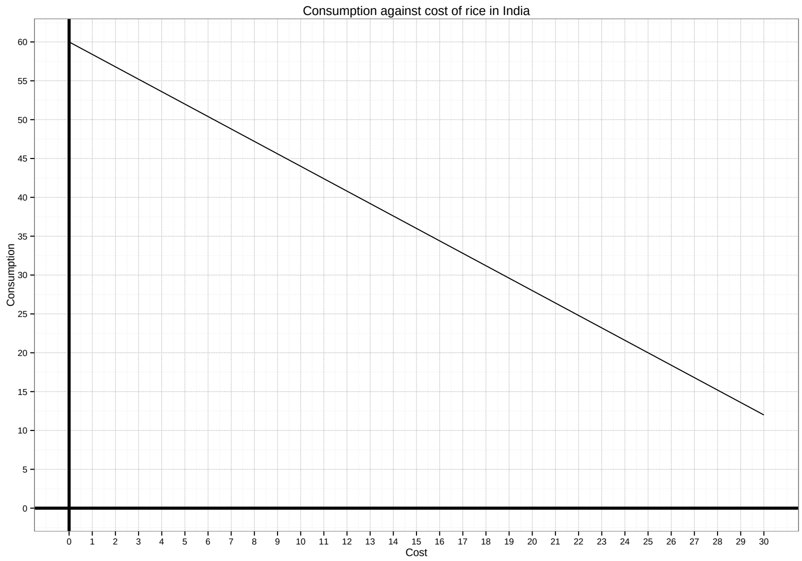


Figure 1.2: Finding the equations of lines.

The graph shows how the United Nations (UN) believes the annual consumption of rice in India (y kilograms per household) changes with the unit cost (x US\$).

(a) Using the graph, obtain a linear function for demand in terms of cost.

(b) Use the graph and your linear function to predict demand for rice if the unit cost of rice is \$22.



(a) We clearly have a linear function, and $c = 60$. The line passes through (12.5,40) and (25,20) and so

$$\begin{aligned}\text{Gradient} &= \frac{40-20}{12.5-25}\\ &= \frac{20}{-12.5}\\ &= -1.6.\end{aligned}$$

Hence we get the linear function of consumption, in terms of cost, as

$$y = -1.6x + 60.$$

(b) If the unit cost of rice is \$22, then demand will be

$$\begin{aligned}y &= -1.6 \times 22 + 60\\ &= 24.8\text{kg}.\end{aligned}$$

We can easily arrive at the same answer from our graph.

1.2.4 Solving linear equations

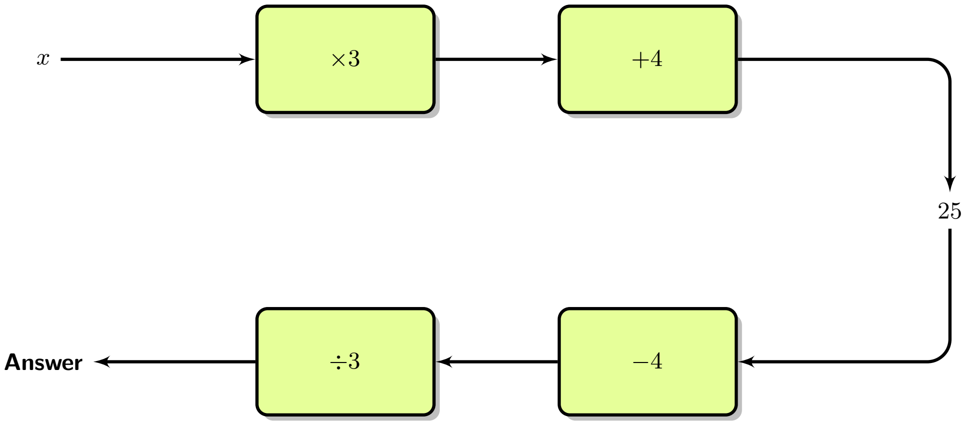
In section 1.2.2 we looked at how we could plot a linear function, and in section 1.2.3 we then looked at how to obtain the equation of a straight line from a graph. In this section we will look at how to solve simple linear equations. Let's return to the example that we saw at the very beginning of this section on linear functions, that is

$$y = 3x + 4.$$

Suppose we knew that $y = 25$; then we have the linear *equation*

$$3x + 4 = 25.$$

How can we work out the value for x? This can be clearly demonstrated using a simple schematic:



Then, when we arrive at the answer we have

$$x = \frac{25 - 4}{3} = 7.$$

More algebraically, we would write:

$$\begin{array}{lr} & 3x + 4 = 25 \\ \text{Subtract 4} & 3x = 21 \\ \text{Divide by 3} & x = 7 \end{array}$$

Solve the following linear equations for x.

1. $5x - 3 = 42$
2. $2(6x + 3) = 186$
3. $\frac{2x + 5}{3} = 7$

The equations we encounter in real–life accounting and finance are often a little more complicated than those we have just looked at – at first glance, anyway. For example, suppose we are required to solve the following equation for x:

$$2(5x + 3) = 6x - 8.$$

Here, we need to expand the brackets on the left–hand–side and then collect together the like terms before solving for x, i.e.:

$$\begin{aligned} 10x + 6 &= 6x - 8 \\ 10x - 6x &= -8 - 6 \\ 4x &= -14 \\ x &= -14 \div 4 \\ &= -3.5 \end{aligned}$$

Consider another example where we need to solve for x:

$$\frac{3}{2x+4} + 5 = 3$$

The first thing that we can do is subtract 5 from both sides giving:

$$\frac{3}{2x+4} = -2$$

Next, we can get rid of the fraction by multiplying by it's denominator, $(2x + 4)$:

$$3 = -2(2x + 4)$$

Then we can easily solve for x, just as we have done in the previous examples:

$$\begin{aligned} 3 &= -4x - 8 \\ 3 + 8 &= -4x \\ 11 &= -4x \\ \text{and so } x &= 11 \div -4 = -2.75 \end{aligned}$$

Solve each of the following linear equations for x and t respectively.

1. $\dfrac{6}{x+4} = 4.$
2. $\dfrac{-4t+4}{5t-6} = 8.$

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A firm manufactures a commodity that costs £20 per unit to produce. In addition, the firm has fixed costs of £2000. Suppose the firm makes x units.

(a) Write down an algebraic expression for the total costs incurred by the firm.

(b) Suppose the firm sells all x units produced, and each unit sells for £75. Write down an expression for the profit made by the firm.

(c) Hence find the number of units that must be produced to meet a target profit of £14,500.

(a) If the firm makes x units, and each unit costs £20 to produce, then the associated costs will be

$$20 \times x.$$

If we now add in the fixed costs of £2000 we get

$$\text{Total Costs} = 20x + 2000$$

(b) Profit will be income from sales minus any costs, i.e

$$\begin{aligned}\text{Profit} &= 75x - (20x + 2000)\\ &= 75x - 20x - 2000\\ &= 55x - 2000.\end{aligned}$$

(c) If the target profit is £14,500, ten

$$\begin{aligned}14500 &= 55x - 2000\\ 14500 + 2000 &= 55x\\ 16500 &= 55x\end{aligned}$$

$$\begin{aligned}x &= \frac{16500}{55}\\ &= 300.\end{aligned}$$

Hence they should produce 300 units.

1.2.5 Finding the equation of a line (revisited)*

In section 1.2.3 we thought about how to find the equation of a straight line, given that we have a graph of that line. We will now consider how to find the equation of a line algebraically, given

(i) the gradient and *one* point that the line passes through,

(ii) *two* points that the line passes through.

Gradient–point method

Suppose that we are told that the gradient of a line is 6. We are also told that the line passes through the point $(3, 25)$. What is the equation of this line?

We are told that $m = 6$, and so we have

$$y = 6x + c.$$

So now we just need to find c. We are also told that the line passes through the point (3,25), i.e. the point where $x = 3$ and $y = 25$. Substituting these values into the equation above gives:

$$\begin{aligned} 25 &= (6 \times 3) + c \\ 25 &= 18 + c. \end{aligned}$$

Solving for c we get $c = 7$, and so we arrive at the equation $y = 6x + 7$ for our line.

Point–point method

Suppose we are told that a line passes through the points (1,3) and (5,35). What is the equation of the line?

Again, we need to find the gradient, m, and the intercept, c. Using the gradient formula from section 1.2.3, we have:

$$\begin{aligned} \text{Gradient} &= \frac{\text{Change in } y}{\text{Change in } x} \\ m &= \frac{35 - 3}{5 - 1} \\ &= \frac{32}{4} \\ &= 8. \end{aligned}$$

So far we have

$$y = 8x + c,$$

and we just need to find c. We know that the line passes through the point (1,3), so we can apply the same procedure as we did in the gradient–point method. That is, substitute $x = 1$ and $y = 3$ into the above equation:

$$\begin{aligned} 3 &= (8 \times 1) + c \\ 3 &= 8 + c \\ 3 - 8 &= c \\ -5 &= c. \end{aligned}$$

So the equation of the line is $y = 8x - 5$. Here it doesn't matter which point we pick to substitute values in for. You can check that you would get exactly the same answer if you substituted in $x = 5$ and $y = 35$. In practice it makes sense to just pick the point that you think will give the easiest calculation.

1.2.6 Solving two linear equations simultaneously*

In real–life accounting and finance problems, we often have more than one equation to solve. You might remember how to solve a pair of linear equations simultaneously from your maths course in school. If not, this section will serve as a reminder. Suppose we want to find the values of x and y that satisfy both of the following equations:

$$2x + 5y = 30 \qquad (1)$$
$$4x + 6y = 48 \qquad (2)$$

Graphical solution

We can plot both equations (1) and (2) on a graph and see where they intersect – at this point, both equations are satisfied by the same x and y values. In order to plot these lines, we *could* rearrange (1) and (2) into the form $y = mx + c$ and then plot the graphs in exactly the same way as before. However, a much quicker way is to obtain the two points at which the lines cut into the x and y axes, and join these two points.

For example, for equation (1):

- When $x = 0$, $5y = 30 \Rightarrow y = 6$, giving the point (0,6)
- When $y = 0$, $2x = 30 \Rightarrow x = 15$, giving the point (15,0)

Similarly for equation (2):

- When $x = 0$, $6y = 48 \Rightarrow y = 8$, giving (0,8)
- When $y = 0$, $4x = 48 \Rightarrow x = 12$, giving (12,0)

Figure 1.3 shows the lines for both equations (1) and (2). It is clear that they intersect at the point (7.5,3), telling us that $x = 7.5$ and $y = 3$ satisfy both of our equations simultaneously. Solving a pair of linear equations simultaneously using a graph is very simple but it is not always the best approach. An accurate graphical solution is sometimes difficult to achieve, depending on the scale you have chosen and how well you have drawn the graph in the first place. If the true solution for x and/or y is a number to more than two decimal places, it can be very difficult to "read off" the correct values from a graph, even if you *have* drawn it very accurately. A better approach in such cases is the algebraic approach, and one such method is known as *elimination*.

Algebraic solution: elimination*

We can also solve simultaneous equation algebraically - one method is "elimination", so–called because we eliminate either the x's or the y's by either:

(i) adding the two equations together, or

(ii) subtracting one from the other.

We will demonstrate this technique using equations (1) and (2). First of all, we need to make the *coefficients* of the x's or the y's the same (you should only do this for one or the other in each problem. It shouldn't be possible to do this for x and y at the same time). Let's choose the x's to eliminate. Notice that if we multiply equation (1) by 2, then the coefficient of x in each equation will be the same. We get,

$$4x + 6y = 48 \qquad (2) \qquad \textit{[Equation(2) is unchanged]}$$
$$4x + 10y = 60 \qquad (3) \qquad \textit{[Equation (1)} \times \textit{2]}$$

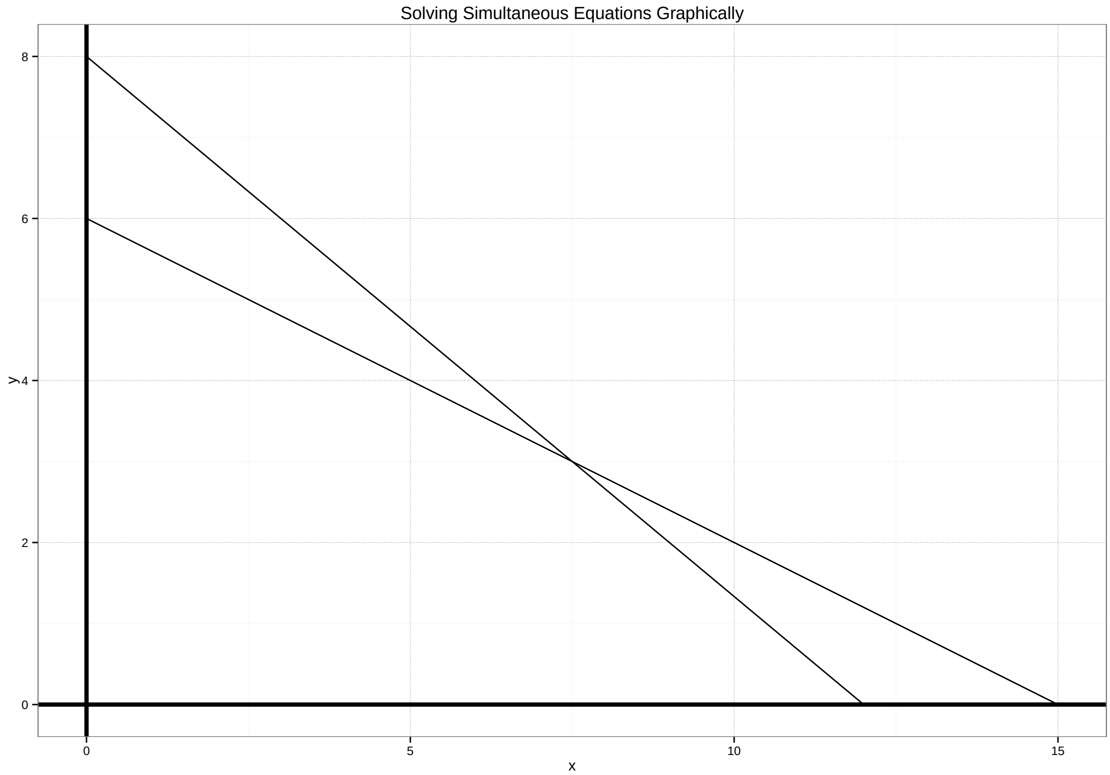

Figure 1.3: Solving simultaneous equations using a graph.

We have re-labelled equation (1), after having multiplied it by 2, as (3); equation (2) has stayed the same. Now if we subtract equation (2) from equation (3) we get

$$\begin{aligned}(4x + 10y) - (4x + 6y) &= (60) - (48) \\ 0x + 4y &= 12.\end{aligned}$$

This has eliminated the x's. Now we know that

$$\begin{aligned}4y &= 12, \qquad \text{and so} \\ y &= 3.\end{aligned}$$

Since we now have a value for y, all we need to do is find the value for x. To do this we can simply substitute $y = 3$ into any of the equations above – (1), (2) or (3) – and then solve for x. For example, substituting $y = 3$ into equation (2) gives:

$$\begin{aligned}4x + 6y &= 48 \\ 4x + (6 \times 3) &= 48 \\ 4x + 18 &= 48 \\ 4x &= 48 - 18 \\ 4x &= 30 \\ x &= 7.5.\end{aligned}$$

So the algebraic solution gives $x = 7.5$ and $y = 3$, exactly the same as the solution we found using the graph. You would get the same answer if we were to first eliminate the y's.

Note:

- If either the x's or the y's already have the same coefficient, there is no need to multiply any of the equations to *make* this happen
- If the signs of the unknown you have chosen to eliminate are the *same* (e.g both + or both −), you would subtract one equation from the other, as we have in this example
- If the signs of the unknown you have chosen to eliminate are *different* (e.g one + and one −), you would add the two equations together
- Be aware of the effect of subtracting negatives
- In this example we only needed to multiply one of our two equations to get one of the coefficients to be the same. It is possible that we may need to multiply *both* by some number to achieve this

Solve the following linear equations simultaneously using the algebraic method (don't use a graph).

$$4x + 8y = 30$$
$$7x - 6y = 5$$

✎

1.3 Quadratic functions

As you will see in the Professional Skills module, linear functions are very useful in accounting and finance applications. However, we will also consider the use of non–linear functions – the simplest of these being *quadratic* functions, where the highest power in x is a 2. For example,

$$y = 3x^2 + 5x - 6$$

is a quadratic function, since the term in x^2 gives the highest power of x. If there was a term in x–cubed, for example $7x^3$, then the function would not be quadratic (in fact, we would call such a function a *cubic*). When plotted, such functions do not give a straight line, and we will study such non–linear functions in detail throughout the module. For now, it would be a good idea to brush up on some basic skills for processing quadratics: in particular, *factorising* quadratics and *solving* quadratics.

1.3.1 Factorising a quadratic*

Factorising a quadratic expression employs the same idea as we saw earlier, in section 1.1.7, but is a little more involved. You should also note that not *all* quadratic expressions can be factorised. Essentially, we want to do the opposite of the "FOIL" method discussed in section 1.1.6. Consider the expression

$$x^2 + 5x + 6.$$

How would we go about factorising it? Well, first, consider the coefficient of the x term and the constant at the end. We want to find two numbers that add together to give the x–coefficient, and multiply to give the constant term. So for this example we want to find two numbers that add to give 5, and multiply to give 6. Obviously, 2 and 3 will work here! Now that we have these we can factorise as follows:

$$x^2 + 5x + 6 = (x + 3)(x + 2).$$

You can check that this gives the original expression by multiplying out the brackets again and making sure that it gives you what you started with.

It is possible that there are negative signs in our quadratic. For example,

$$x^2 + 2x - 3 = (x + 3)(x - 1).$$

We needed two numbers that, when multiplied together, would give –3, but when added together would give 2. Now, $-1 \times 3 = -3$, but also $1 \times -3 = -3$. However, the second pair cannot work, as $1 + (-3) = -2$, and our x–coefficient is $+2$; hence the use of –1 and 3 in the factorisation above.

In general there are a number of ways we could think about factorising a quadratic, a guide for which is given in table 1.2:

$x^2 + ax + b$	$x^2 + ax - b$ or $x^2 - ax - b$	$x^2 - ax + b$
$(x+?)(x+?)$	$(x+?)(x-?)$	$(x-?)(x-?)$

Table 1.2: Factorising quadratics.

In practice, experience tells us which sign goes in front of which number, and practice makes perfect! In more complicated examples, we may also have a coefficient in front of the x^2 term; again, the same process applies but now inside each of our brackets we may need a coefficient of x when we factorise. This is perfectly fine, and experience and practice will help in putting all numbers and signs in the correct places. You can always check your answer by expanding the brackets back out again (using "FOIL") and checking that you get the same as what you started with.

Factorise the following quadratics:

(a) $x^2 - 6x - 7$

(b) $3x^2 + 7x + 2$

(a) If we use the table we can already say that the solution must have the form

$$(x + ?)(x - ?).$$

Let's think about what numbers we can multiply together to give 7. Well, there is only one choice really: 1 and 7. So we now know that our solution will be either

$$(x + 7)(x - 1) \quad \text{or} \quad (x + 1)(x - 7).$$

But which is correct? Well we want to add to give -6, so using the 7 and 1 this only makes sense if we do $-7+1$, otherwise we would get $+6$, so our solution is

$$(x+1)(x-7).$$

We could check by expanding the brackets.

(b) This one is a little more difficult until you get the hang of quadratics, but the same rules still apply. From the table we know that both the signs in the brackets must be + signs. The numbers at the ends have to multiply together to give 2, but this time we have to have the coefficients of x multiply to give 3. We could only have 1×3 as an option so our solution will look something like

$$(3x+?)(x+?).$$

And now since the last 2 numbers have to multiply to give 2, so this must be a 1 and a 2. So either we have

$$(3x+1)(x+2) \qquad \text{or } (3x+2)(x+1).$$

Thinking about how "FOIL" works, the $3x$ will multiply the number in the other bracket, and we want $7x$ in total. We will either have $3x \times 1 + x \times 2$ or $3x \times 2 + x \times 1$. To make $7x$, we need to use the second of these, so our solution is

$$(3x+1)(x+2).$$

1.3.2 Solving quadratics*

In many applications it will be necessary to be able to *solve* quadratic equations. This typically involves solving an equation of the form:

$$ax^2 + bx + c = 0.$$

This is the general form of a quadratic equation. There are a number of ways to solve such quadratics but for the moment we are principally interested in just two of them – solution by factorisation and solution by the quadratic formula. All quadratics have 2 solutions – this is true because the highest power of x is 2. In fact, for any polynomial there exists a number of solutions equal to the highest power of x.

Solution by factorisation

We have already looked at how to go about factorising a quadratic in section 1.3.1. Suppose now we have

$$x^2 + 5x + 6 = 0.$$

To solve this quadratic equation (it's now an equation because the quadratic has been set equal to something!) we need to find all the values of x that make this equation true. We are going to use factorisation to solve this, so let us now factorise the left hand side, as before, to arrive at the equivalent equation

$$(x+3)(x+2) = 0.$$

You should know, by now, that when you multiply anything by zero, you always get zero. In our example, when we multiply $(x+3)$ by $(x+2)$ we get zero and so one of the terms inside the brackets – or perhaps both – must be zero; that is

$$x+3=0, \qquad x+2=0.$$

Solving both of these will give us all of the values of x which then make the quadratic equal to 0, solving the problem! It is fairly trivial to see that we get the two solutions:

$$x = -3, \qquad x = -2.$$

We can check that these indeed are the solutions by substituting into our original equation:

$$x^2 + 5x + 6 = 0$$

$$\begin{aligned} &= (-3)^2 + 5 \times (-3) + 6 \\ &= 9 - 15 + 6 \\ &= 0 \end{aligned} \qquad \begin{aligned} &= (-2)^2 + 5 \times (-2) + 6 \\ &= 4 - 10 + 6 \\ &= 0 \end{aligned}$$

Solution by formula

As mentioned earlier, it is not always possible to factorise a quadratic expression directly. If this is the case, we need some way of solving a quadratic equation that doesn't require factorisation. Luckily for you, we have the "quadratic formula". For a general quadratic equation:

$$ax^2 + bx + c = 0,$$

with any coefficient values, a, b and c, we can find both solutions x using

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

The part under the $\sqrt{}$ sign is called the discriminant, often abbreviated as D. We can use this to determine whether we will get two real solutions, one repeated real solution or two complex solutions to our quadratic using table 1.3. In our examples, we will never see $D < 0$.

Value of $D = b^2 - 4ac$	Type of solution
> 0	Two real solutions
$= 0$	One, repeated real solution
< 0	Two complex solutions

Table 1.3: Determining the type of solution to a quadratic equation using the discriminant.

Use the discriminant to determine what type of solutions we will get from the following quadratic. Then use the quadratic formula to *find* these solutions.

$$x^2 + 5x + 6 = 0$$

We can determine what type of solution we expect by using the discriminant. Comparing with the general quadratic equation our coefficients here are $a = 1$, $b = 5$ and $c = 6$. Thus,

$$\begin{aligned} D &= b^2 - 4ac \\ &= 5^2 - 4(1)(6) \\ &= 25 - 24 \\ &= 1 \\ &> 0. \end{aligned}$$

Now we know that we are expecting two real solutions (we know that this must be correct since we have already solved this particular equation in the previous section). OK, well what are our solutions? We can now use the quadratic formula to find what values of x solve our equation.

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-5 \pm \sqrt{1}}{2}. \end{aligned}$$

This leaves us with:

$$\begin{aligned} x &= \frac{-5+1}{2} \\ &= \frac{-4}{2} \\ &= -2 \end{aligned} \qquad \begin{aligned} x &= \frac{-5-1}{2} \\ &= \frac{-6}{2} \\ &= -3. \end{aligned}$$

Notice that these are the exact same solutions that we got when we solved via factorisation. The quadratic formula will *always* work to give us the solution of any quadratic equation with $D \geq 0$.

For the following quadratics: (i) find the discriminant; (ii) determine what type of solutions the quadratics will give; (iii) find the solutions, if possible.

1. $x^2 + 4x + 4$
2. $2x^2 + 7x + 1$
3. $x^2 + x + 2$

1.4 Further practice questions

These questions are to supplement the material presented in this chapter. They will give you good practice at the stuff we've covered so far. Full solutions are given to all of these questions in section 3.1 towards the end of this booklet.

1. What are the intercept and gradient of the following linear functions?

   (a) $y = 2x + 5$

   (b) $y = 12 - 2x$

   (c) $8y = 2x + 16$

   (d) $y = x$

2. Put the linear functions in question 1 in order of "steepness", from most steep to least steep.

3. A recent British Gas investigation examined factors which influenced the length of time it takes customers to pay their utility bills (see the British Gas website for details about this investigation and a fully download-able report). Their report revealed that the size of a customer's bill (£x) was the most important factor influencing the time it took customers to pay this bill (y, in days).

   In particular, British Gas express the time it takes a customer to pay their bill as a linear function of the size of their bill:
   $$y = 0.06x + 7.44.$$

   (a) What are the intercept and gradient of this linear function?

   (b) If Amy Hall's gas bill was £450, how long can we expect it will take for her to pay this bill?

   (c) Suppose it took Ryan Bodsworth 10 days to pay his gas bill. Solve the above linear equation for x to find out how much Ryan's bill was.

4. Joe Wildman needs a new carpet for his bedroom. His room is rectangular in shape, and the length of the room is 3 times the width. Let x be the width of the room, in metres.

   (a) Write down an expression for the perimeter of Joe's bedroom. Simplify this expression as far as possible.

   (b) Write down an expression for the area of Joe's bedroom.

   (c) Which of your expressions in (a) and (b) is *linear*?

5. Daniel Eastwood is an entrepreneur who has recently developed a miracle hangover cure. The number of hours until a hangover is cured (y) is thought to be related to the dose given (x mg).

   Specifically, if a person suffering with a hangover is not given any of the drug at all, the hangover will pass within 10 hours. However, a 50mg dose should see the hangover pass within 2 hours.

   (a) Assuming y is a linear function of x, find the intercept, c, and the gradient m. Hence write down the linear function.

   (b) Assuming side effects are negligible regardless of the dose, what is the optimal dose to give hangover sufferers?

Chapter 2

Some basic statistics skills

2.1 Types of data

In the Professional Skills module, you will be introduced to various methods of data analysis. However, the type of analysis you use will often depend on the sort of data you have. Data can be one of two types: *qualitative* or *quantitative*. Qualitative variables have non–numeric outcomes. They are usually *categorical*. Examples of qualitative variables include: sex of a person or animal, colour of a car, mode of transport and football team supported. Quantitative variables have numeric outcomes with a natural ordering. Examples include: people's height, time to failure of a component and number of defective components in a batch.

Quantitative data are usually one of two types: *discrete* or *continuous*. Discrete data can only take a sequence of distinct values (which are usually the integers, although not necessarily so). Discrete variables can be countable – for example, the number of defective pieces in a manufacturing batch, the number of people in a tutorial group, or a person's shoe size. However, there are other kinds of discrete data, such as *ordinal* data. Ordinal data are ordered discrete data, where the outcomes are not really numbers in the usual sense. For example, if you are asked to rank a response to a question between 1 and 10, from strongly disagree to strongly agree, an answer of 8 obviously indicates stronger agreement than one of 4, but not necessarily twice as strong in any meaningful sense. Continuous data can take any value over some continuous scale – for example: height, weight, time taken to be served in a bank, etc. Continuous data can take any value over some continuous scale – for example, height, weight, length of time spent in a queue, etc.

2.2 Descriptive statistics

There is a whole host of descriptive statistics available for summarising data. This section of the booklet will remind you of some of the simplest.

2.2.1 Measures of average

The idea of a measure of average is to give us a single number which is representative of our dataset as a whole. There are lots of different measures of average – some of which might be more useful than others, depending on the scenario. Here, we provide a brief description of the most commonly–used. For the purpose of demonstrating each calculation, please refer to the set of data in table 2.1, which shows monthly ice cream sales for an ice cream shop in Newcastle.

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
x (sales in £000's)	2	5	8	11	14	16	15	10	9	9	8	8

Table 2.1: Ice cream sales data for use in sections 2.2.1 and 2.2.2.

Arithmetic mean

The arithmetic mean of a set of data is simply what you get by adding all of the observations together and dividing by how many you have. In statistics the arithmetic mean of some data x is denoted by putting a bar over the letter, i.e. $\bar{x}$, read aloud as "x bar". So, for our data set:

$$\begin{aligned}\bar{x} &= \frac{2+5+8+11+14+16+15+10+9+9+8+8}{12} \\ &= \frac{1}{12} \times 115 \\ &= \frac{115}{12} \\ &= 9.58\dot{3},\end{aligned}$$

i.e. about £9,583 in sales over the year.

More algebraically we could write

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i,$$

which means the same as the description for calculating the arithmetic mean given in words above. That is, for each of the n data points, x_i, $i = 1, \ldots, n$, we add them up ($\sum$), then divide by how many there are (n). In our example $n = 12$, $x_1 = 2, x_2 = 5, \ldots, x_{12} = 8$. If you're not sure about this notation, don't worry – as long as you know how to do the calculation!

Mode

The mode of a data set is simply the value in the data that occurs most often. It is possible to have more than one mode if more than one value has the same (but largest) number of repetitions. In our example dataset we can see that 8 has the most repetitions (there are 3 of them, more than any other value). So the mode of this data set is 8, or £8,000.

Median

The median of a data set is the middle value. In some cases – for example, if there is one data point far from the rest – the median may be a more representative measure of average than the mean, as the mean can be distorted by very large (or very small) values. To find the median, it is easiest to sort the data into ascending order:

x	2	5	8	8	8	9	9	10	11	14	15	16

You can then can 'cross out' numbers from both ends simultaneously until you arrive at the middle:

~~2~~ ~~5~~ ~~8~~ ~~8~~ ~~8~~ 9 9 ~~10~~ ~~11~~ ~~14~~ ~~15~~ ~~16~~

Here you see that we have two numbers remaining. If we have an odd number of data points this would not be a problem, as only one would be left after crossing out from both ends – and this would be the median. If we have an even number of data points, like here, we have two remaining

94	115	109	114	98	104	124	105	94	95
102	104	108	99	97	94	100	93	99	112
92	94	101	104	107	103	107	106	112	112
116	78	80	99	106	89	100	91	85	107
103	111	106	86	93	114	93	87	106	116
92	100	99	96	93	120	102	103	103	106
105	100	98	96	104	96	82	96	111	87
107	109	85	99	108	90	115	100	97	94
106	108	95	111	99	106	102	101	104	88
97	106	104	108	109	99	122	94	103	95

Table 2.2: A data set of the number of cars passing a motorway bridge during rush hour over 100 days.

values that haven't been crossed out. In this scenario, the median is the number that is half way between them. In this instance both of our numbers are 9; half way between 9 and 9 is still 9, so the median of our data is 9, or £9,000 in sales over the year.

2.2.2 Measures of spread

You have probably already come across two measures of spread, although you may not have realised they were "measures of spread"! These are the range, and the inter–quartile range. Consider the two datasets below:

Dataset 1	21	22	23
Dataset 2	12	22	32

Both datasets have the same mean and median (mean = median = 22 in both cases), but both datasets are very different: dataset 2 is much more tightly concentrated around the middle value than dataset 1, which is much more "spread out". Thus, an average alone would not describe these datasets very well; we also need a measure of spread.

The range is just the largest value minus the smallest value, giving:

$$\begin{aligned} \text{Range}_1 &= 23 - 21 = 2 \\ \text{Range}_2 &= 32 - 12 = 20 \end{aligned}$$

So from the range we can see that dataset 2 is much more dispersed than dataset 1. We will come back to measures of spread in the Professional Skills module, where we will also consider the inter–quartile range and the standard deviation. Incidentally, the range for the ice cream sales data is $16 - 2 = 14$, or £14,000.

2.3 Graphical summaries

You may remember from your studies at school that there are a number of ways of representing data graphically. Graphics are a good way of visualising data, and understanding patterns and relationships in datasets. For the first example let us consider the dataset in table 2.2.

2.3.1 Stem and leaf plots

A stem and leaf plot is a nice way of representing data. It shows the shape of the distribution whilst at the same time showing the "raw" data values themselves. An example of a stem and leaf plot for the cars dataset is shown in table 2.3.

If you look at the stem and leaf plot "sideways" you can see that it has a bar chart–type shape, but we can still tell what all of the individual data values are. Here, each value in the stem represents "tens" and the leaves represent "units" (e.g. 9|5 represents 95: 9 "tens" and 5 "units"). There are no rules as to how to pick the stem and leaf units, other than something sensible should be used. For this example we have two separate branch values for numbers between, say 90–94, and 95–99. This makes sense here as we have a large dataset; otherwise, the number of leaves on each "branch" could be too long (to fit on the page, perhaps!).

8	134
8	55556678899
9	00111223333344444
9	5555666777777888888899999
10	000001122234444
10	55555778899
11	000122344
11	5789
12	1123

Table 2.3: A stem and leaf plot of the cars data from table 2.2

2.3.2 Bar charts

Let us now consider the data set in table 2.4 which shows the numbers of different vehicles travelling under the motorway bridge during rush hour.

Vehicle	Frequency
Car	76
Lorry	40
Motorbike	17
Coach	6
Caravan	3
Total	142

Table 2.4: Numbers of different vehicles travelling under a motorway bridge during rush hour.

An obvious choice for such *categorical* data – where we have frequencies within distinct categories – is the good old bar chart. Figure 2.1 shows a simple bar chart of these data. The values up the side represent the frequencies within each group. Notice that the bars are distinct – that is, there are clear gaps between them to emphasise the different categories. Also notice that no colour has been used – perhaps using colour could have made the chart more interesting?

2.3.3 Pie charts

The final graphical tool that we will look is the pie chart. The pie chart can be very useful for getting a good understanding of the proportion of the whole dataset that is in each group; again, these charts are useful for displaying categorical data. The full circle in the pie chart (shown in figure 2.2 for the vehicle data) represents 100% of the data. The segments represent the proportions of each group of the dataset. When drawing these by hand, you need to work out the angle subtended for each category, as a proportion of the 360° in the circle as a whole. For example, for "cars" we would have

$$\text{Angle subtended} = \frac{76}{142} \times 360^{\text{o}} = 192.7^{\text{o}}.$$

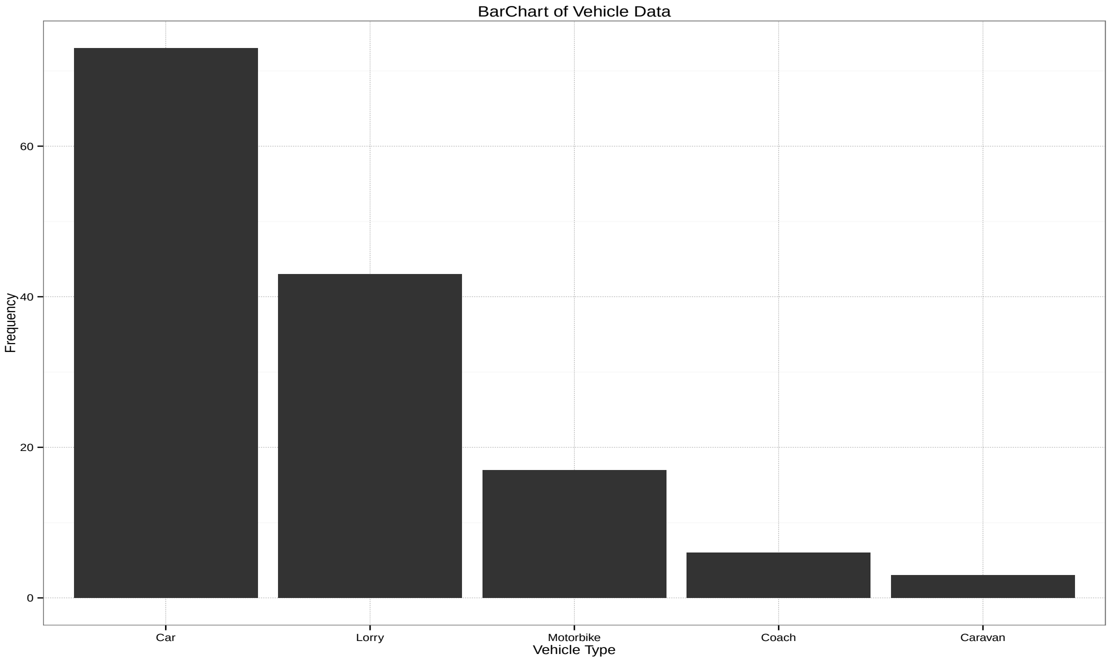

Figure 2.1: A bar chart of the vehicle data from table 2.4.

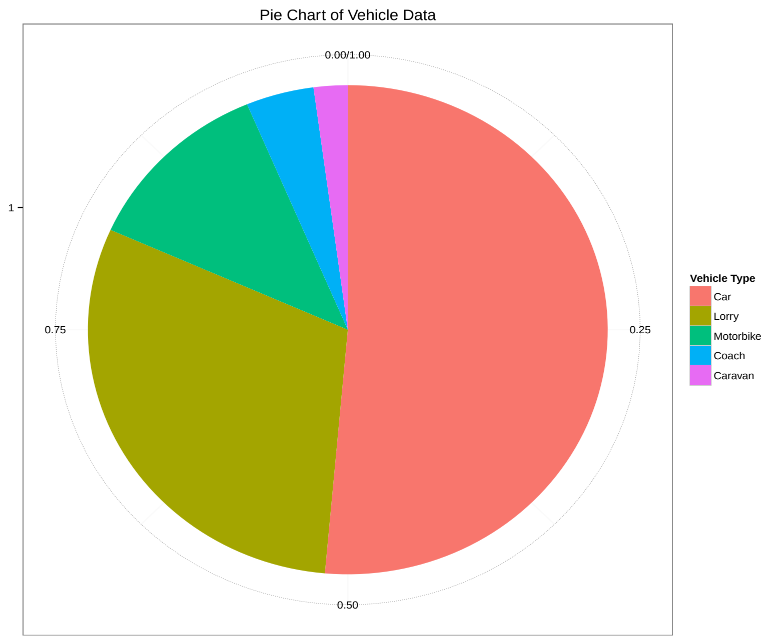

Figure 2.2: A pie chart of the vehicle data in table 2.4.

2.4 Further practice questions

These questions are to supplement the material presented in this chapter. They will give you good practice at the stuff we've covered so far. Full solutions are provided to all of these questions in Section 3.2.

1. Calculate the mean, median and mode for the following data.

9	15	12	13	11
9	13	6	11	11
12	17	10	10	14
9	9	10	16	11
7	12	11	11	11

Table 2.5: Number of people in a banks queue at lunch time over 5 weeks.

2. From the data in Question 1, produce an appropriate graphical summary.

3. The following table shows the numbers of people from different continents that answered a recent online survey. Which graphical summary/summaries might be useful to show these data and why? Sketch an appropriate graph.

Africa	The Americas	Asia	Australia/Oceania	Europe
79550	90272	101805	82333	115945

Table 2.6: Respondees to an online suvey by continent

Chapter 3

Solutions to practice questions

3.1 Solutions to basic maths questions

1. (a) $m = 2$ and $c = 5$
   (b) $m = -2$ and $c = 12$
   (c) Dividing through by 8 gives
   $$y = \frac{1}{4}x + 2,$$
   so $m = \frac{1}{4}$ and $c = 2$
   (d) $m = 1$ and $c = 0$

2. Order of "steepness": (a) and (b), (d), (c)

3. (a) $m = 0.06$ and $c = 7.44$
   (b) For Amy, we have $x = 450$, giving
   $$y = 0.06 \times 450 + 7.44 = 34.44,$$
   or 35 days.
   (c) For Ryan we have $y = 10$, giving
   $$\begin{aligned} 10 &= 0.06x + 7.44 \\ 10 - 7.44 &= 0.06x \\ 2.56 &= 0.06x. \end{aligned}$$
   Dividing through by 0.06 gives $x = £42.67$.

4. The width of Joe's bedroom is x metres and so the length is $3x$ metres.
   (a) The perimeter P is given by
   $$P = 3x + 3x + x + x = 8x$$
   (b) The area A is given by
   $$A = 3x \times x = 3x^2$$
   (c) Only the expression for P is linear, the expression for A includes a term in x^2.

5. (a) We are told that if a person is not given any of the hangover medicine at all, then we can expect a hangover to last for 10 hours. Thus the y-intercept must be $c = 10$, as this corresponds to the point with co–ordinates (0,10). We are also told that the point (50,2) lies on the line and so

$$\begin{aligned}\text{Gradient} &= \frac{\text{Change in } y}{\text{Change in } x}\\ &= \frac{10-2}{0-50}\\ &= \frac{8}{-50}\\ &= 0.16.\end{aligned}$$

So the linear function is $y = -0.16x + 10$.

(b) The optimal dose would cure the hangover immediately, i.e after zero hours. Thus at this point $y = 0$, giving,

$$\begin{aligned}0 &= -0.16x + 10\\ 0 - 10 &= -0.16x\\ -10 &= -0.16x\\ \\ x &= \frac{-10}{-0.16}\\ &= 62.5\text{mg}\end{aligned}$$

3.2 Solutions to basic statistics questions

1. The descriptive statistics for the data set are:

	Value
Mean	11.2
Median	11
Mode	11

2. We could use a stem and leaf plot for this set of data:

0	679999
1	0001111111222334
1	567

Table 3.1: Stem and leaf plot for data in Question 1.

3. The descriptive statistics for the dataset are:

	Value
Mean	93981
Median	90272

We could use either a bar chart or a pie chart for this data as shown in figure 3.1.

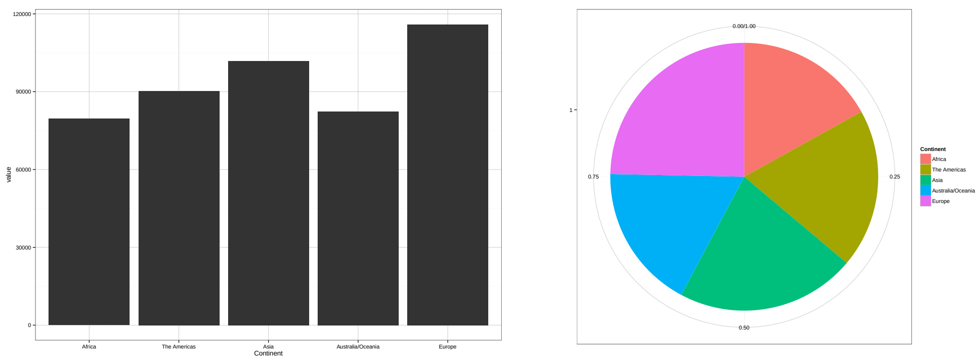

Figure 3.1: Graphical summaries of the survey data.