

NEWCASTLE UNIVERSITY

SCHOOL OF MATHEMATICS & STATISTICS

SEMESTER 2 2008/2009

MAS1403

Quantitative Methods for Business Management

Time allowed: 2 hours

Credit will be given for ALL answers to questions in Section A, and for the best TWO answers to questions in Section B. No credit will be given for other answers and students are strongly advised not to spend time producing answers for which they will receive no credit.

Marks for each question are indicated. However you are advised that marks indicate the relative weight of individual questions, they do not correspond directly to marks on the University scale.

There are FIVE questions in Section A and FOUR questions in Section B.

Answers to questions in Section A should be entered directly on this question paper in the spaces provided. Rough work should be done on the blank sides of the pages. The rough work will not be marked. This question paper must be handed in, attached inside an anonymised cover sheet, at the end of the examination.

Calculators may be used. Graph paper will be provided. This examination is "open-book"; students may consult their notes, text books etc during the examination.

SECTION A

- A1.** A business accountancy firm is conducting an audit of executive salaries for one of its customers. The salaries of ten executives are shown below (in thousands of pounds).

45 48 52 55 57 60 66 70 75 82

- (a) In the space below, construct a stem-and-leaf plot for this sample of executive salaries, and comment on the shape of the distribution.

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Stem unit =

Leaf unit =

- (b) Calculate the mean and standard deviation for these salaries, and construct a 95% confidence interval for the population mean executive salary for this company.

Answer:

[10 marks]

- A2.** The *Eco Coffee Shop* produces bags of organic coffee with a mean weight of 500g. The company decides that all bags weighing less than 470g *or* more than 510g will be repacked. Assuming the weight of these bags is Normally distributed with standard deviation 15g, find the percentage of bags that will be repacked.

Answer:

[9 marks]

A3. During the “Early–Bird Specials” period, customers arrive at *Pacino’s* Italian restaurant at a rate of three every twenty minutes. The “Early–Bird” period runs from 6pm–7pm.

- (a) What probability distribution might be reasonable to use to model X , the number of customers who arrive at *Pacino’s*, during the “Early–Bird Specials” period?

Answer:

- (b) Write down the expected number and standard deviation of X .

Answer:

- (c) Find the probability that exactly ten customers arrive during the “Early–Bird Specials” period.

Answer:

- (d) Let Y be the number of customers who arrive at *Pacino’s* over the course of the entire evening (6pm–11pm). Guiseppe, the manager at *Pacino’s*, assumes that $Y \sim \text{Poisson}(45)$. Suggest why Guiseppe’s assumption might be inappropriate.

Answer:

[8 marks]

- A4. 100 passengers on board a *Qantas* flight from Sydney to Brisbane were randomly selected to take part in a customer survey. One of the questions asked:

IN WHICH CABIN DO YOU NORMALLY FLY WITH QANTAS?

The results are shown below.

	First Class	Business Class	Economy	Total
Male	6	11	46	63
Female	9	11	17	37
Total	15	22	63	100

Is there any evidence to suggest that cabin type is associated with gender?

Answer:

[12 marks]

- A5.** Before launching a new product, small businesses usually conduct market research to assess the viability of this new product. As a business analyst, you are interested in whether or not there is a difference in the amount spent on such market research by small companies in northeast and southeast England. From a random sample of small businesses in both regions, the table below summarises the amount spent (in thousands of pounds):

Region	Sample size	Mean	Standard deviation
Northeast	15	86.4	5.8
Southeast	12	90.5	4.5

Perform an appropriate hypothesis test to determine whether there is a difference in the average amount small businesses spend on market research between the regions.

Answer:

[11 marks]

SECTION B

B6. *Bright Sparks* electrical company produces circuit boards to be used in plasma TVs.

- (a) The company's quality inspector decides to check for defective circuit boards. She inspects every 5th circuit board off the production line. What type of sampling scheme is she using?
- (b) Out of the 50 circuit boards that she inspects, 8 are defective. Estimate the probability of a circuit board being defective.
- (c) *Bright Sparks* sell circuit boards to manufacturers of these plasma TVs in batches of four. For a batch of four such circuit boards, use the probability you estimated in part (b) to find $P(X = 0)$, $P(X = 1)$, $P(X = 2)$, $P(X = 3)$, and $P(X = 4)$, where X is the number of defective circuit boards in a batch. What probability model have you used here?
- (d) Every *Bright Sparks* employee gets a bonus of £10 for each batch of 'perfect' circuit boards that are sold (i.e. no defectives in the batch at all). Other employee bonuses are shown in the table below; note that a 'negative bonus' is applied for batches with at least two defective circuit boards.

No. of defectives (X)	Bonus (£)	Frequency (2008)
0	10	200
1	5	154
2	-10	38
3	-40	7
4	-100	1

Using the probabilities you found in part (c), and the bonuses shown above, calculate the *Expected Monetary Value* for this bonus scheme.

- (e) The table above also shows the number of manufacturers (out of 400 in total) who received a batch of circuit boards from *Bright Sparks* in 2008 with $X = 0, 1, \dots, 4$ defectives. Perform a goodness-of-fit test to see if the data observed in 2008 are consistent with the model you used in part (c). [Hint: Use the probabilities in part (c) to obtain your expected frequencies for this test]

[25 marks]

- B7.** In October 2008 model and celebrity Kate Moss endorsed the latest *Top Shop* women's clothing range, where she starred in an advertising campaign aimed at increasing sales (see pictures below – © *TopShop* 2008). Total sales of women's clothing at a selection of *Top Shop* stores are summarised in the table underneath – for periods *before* and *after* this advertising campaign.



Total sales (X thousand pounds)	Percentage		Cumulative percentage	
	Before	After	Before	After
$10 \leq x < 14$	12	2		
$14 \leq x < 18$	32	15		
$18 \leq x < 22$	40	22		
$22 \leq x < 26$	10	38		
$26 \leq x < 30$	5	17		
$30 \leq x < 34$	1	4		
$34 \leq x < 38$	0	2		

- Write down the modal class for sales in each of the before and after periods.
- Calculate cumulative percentages for sales in the periods before and after this advertising campaign.
- Using the cumulative percentages obtained in part (a), construct ogives (cumulative relative frequency polygons) for sales in the before and after periods, and overlay these on the same graph (you should use graph paper here).
- Using your ogives in part (c),
 - estimate the median level of sales for both the before and after periods;
 - estimate the inter-quartile range for sales in the before and after periods.
- Using the ogives in part (c) and the summaries in parts (a) and (d), compare and contrast sales at *Top Shop* before and after the advertising campaign.
- Use the percentages given in the table to estimate mean sales in the after period (to the nearest £). Why is your answer an *approximation* of the sample mean?
- The standard deviation for sales in the after period can be estimated from the above table as £4,819. Assuming that sales in the after period follow a Normal distribution with a mean as calculated in part (f), find the probability that total sales for a randomly chosen *Top Shop* store exceed £28,000.

[25 marks]

- B8. You are the manager of a team of travel consultants based in the holiday resort of Tarifa in southern Spain. The total number of tourists visiting this resort in each of the “Spring” (February–May), “Summer” (June–September) and “Winter” (October–January) seasons over the last three years is shown in the table below (figures are in thousands).

	Spring	Summer	Winter
2006	245	285	150
2007	220	250	120
2008	190	230	105

- (a) On graph paper produce a time series plot for these data, and comment.
 (b) Calculate the moving averages for these data, and overlay these on your plot in part (a).
 (c) Let T represent time, and let time-points t_i , $i = 1, 2, \dots$, correspond to Spring (2006), Summer (2006), ... (respectively). Also, let y_i be the moving average at time t_i , as calculated in part (b). Then we have the following summaries:

$$\begin{aligned} \sum_{i=2}^8 t_i &= 35 & \sum_{i=2}^8 y_i &= 1393.33 \\ \sum_{i=2}^8 t_i^2 &= 203 & \sum_{i=2}^8 t_i y_i &= 6723.33 \end{aligned}$$

Use the summaries above to estimate the linear trend

$$Y = \alpha + \beta T + \epsilon.$$

- (d) The (unadjusted) seasonal effects are given in the table below.

Season	Seasonal effects
Spring	+2.73
Summer	+48.57
Winter	-75.60

Obtain the *adjusted* seasonal effects.

- (e) As the manager of this team of travel consultants you often have to make difficult decisions. You decide that staffing levels used in previous years for the Summer season can only be maintained if the number of tourists in Tarifa remains above 200,000. Use the linear trend equation in part (c), and the adjusted seasonal effects in part (d), to forecast the total number of visitors to this resort this summer (2009). Will you maintain staffing levels used in previous summers this year?

[25 marks]

- B9.** The *Monster Party Company* produce two types of party pack. Their “Ghastly” party pack contains 10 balloons and 64 sweets. Their “Devilish” party pack contains 20 balloons and 16 sweets. The company has 3000 balloons and 8000 sweets available. Each week, the company sells at least 50 of each type of party pack; in total, they make and sell x “Ghastly” party packs and y “Devilish” party packs. The company sells each “Ghastly” party pack at a profit of £1.20 and each “Devilish” party pack at a profit of £1.80.
- (a) Formulate the *Monster Party Company*’s situation as a linear programming problem.
 - (b) On graph paper draw a suitable diagram to enable the problem to be solved graphically, indicating the feasible region and the direction of the objective line.
 - (c) Using your diagram, find the company’s minimum and maximum weekly profit, £ P .
 - (d) Now solve this problem algebraically to verify the company’s maximum weekly profit obtained in part (c).

[25 marks]

THE END

Statistical tables overleaf →

Statistical tables

	One-tailed test Two-tailed test	p				
		10% 20%	5% 10%	2.5% 5%	1% 2%	0.5% 1%
ν	1	3.078	6.314	12.706	31.821	63.657
	2	1.886	2.920	4.303	6.965	9.925
	3	1.638	2.353	3.182	4.541	5.841
	4	1.533	2.132	2.776	3.747	4.604
	5	1.476	2.015	2.571	3.365	4.032
	6	1.440	1.943	2.447	3.143	3.707
	7	1.415	1.895	2.365	2.998	3.449
	8	1.397	1.860	2.306	2.896	3.355
	9	1.383	1.833	2.262	2.821	3.250
	10	1.372	1.812	2.228	2.764	3.169
	11	1.363	1.796	2.201	2.718	3.106
	12	1.356	1.782	2.179	2.681	3.055
	13	1.350	1.771	2.160	2.650	3.012
	14	1.345	1.761	2.145	2.624	2.977
	15	1.341	1.753	2.131	2.602	2.947
	16	1.337	1.746	2.120	2.583	2.921
	17	1.333	1.740	2.110	2.567	2.898
	18	1.330	1.734	2.101	2.552	2.878
	19	1.328	1.729	2.093	2.539	2.861
	20	1.325	1.725	2.086	2.528	2.845
	21	1.323	1.721	2.080	2.518	2.831
	22	1.321	1.717	2.074	2.508	2.819
	23	1.319	1.714	2.069	2.500	2.807
	24	1.318	1.711	2.064	2.492	2.797
	25	1.316	1.708	2.060	2.485	2.787
	26	1.315	1.706	2.056	2.479	2.779
	27	1.314	1.703	2.052	2.473	2.771
	28	1.313	1.701	2.048	2.467	2.763
	29	1.311	1.699	2.045	2.462	2.756
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
	∞	1.282	1.645	1.960	2.326	2.576

Table 1: This table contains values of t for which $\Pr(T > t) = p$, where $T \sim t_\nu$. In a two-tailed test, the tabulated values correspond to $\Pr(|T| > t) = p$

		p				
		50%	10%	5%	1%	0.1%
ν	1	0.45	2.17	3.84	6.63	10.83
	2	1.39	4.61	5.99	9.21	13.82
	3	2.37	6.25	7.82	11.34	16.27
	4	3.36	7.78	9.49	13.28	18.47
	5	4.34	9.24	11.07	15.09	20.52
	6	5.35	10.64	12.59	16.81	22.46
	7	6.35	12.02	14.07	18.48	24.32
	8	7.34	13.36	15.51	20.09	26.13
	9	8.34	14.68	16.92	21.67	27.88
	10	9.34	15.99	18.31	23.21	29.59
	12	11.34	18.55	21.03	26.22	32.91
	15	14.34	22.31	25.00	30.58	37.70
	20	19.34	28.41	31.41	37.57	45.32
	25	24.34	34.38	37.65	44.31	52.62
	30	29.34	40.26	43.77	50.89	59.70

Table 2: This table contains values of x for which $\Pr(X^2 > x) = p$, where $X^2 \sim \chi_\nu^2$

z	-0.09	-0.08	-0.07	-0.06	-0.05	-0.04	-0.03	-0.02	-0.01	0.00
-2.9	0.0014	0.0014	0.0015	0.0015	0.0016	0.0016	0.0017	0.0018	0.0018	0.0019
-2.8	0.0019	0.0020	0.0021	0.0021	0.0022	0.0023	0.0023	0.0024	0.0025	0.0026
-2.7	0.0026	0.0027	0.0028	0.0029	0.0030	0.0031	0.0032	0.0033	0.0034	0.0035
-2.6	0.0036	0.0037	0.0038	0.0039	0.0040	0.0041	0.0043	0.0044	0.0045	0.0047
-2.5	0.0048	0.0049	0.0051	0.0052	0.0054	0.0055	0.0057	0.0059	0.0060	0.0062
-2.4	0.0064	0.0066	0.0068	0.0069	0.0071	0.0073	0.0075	0.0078	0.0080	0.0082
-2.3	0.0084	0.0087	0.0089	0.0091	0.0094	0.0096	0.0099	0.0102	0.0104	0.0107
-2.2	0.0110	0.0113	0.0116	0.0119	0.0122	0.0125	0.0129	0.0132	0.0136	0.0139
-2.1	0.0143	0.0146	0.0150	0.0154	0.0158	0.0162	0.0166	0.0170	0.0174	0.0179
-2.0	0.0183	0.0188	0.0192	0.0197	0.0202	0.0207	0.0212	0.0217	0.0222	0.0228
-1.9	0.0233	0.0239	0.0244	0.0250	0.0256	0.0262	0.0268	0.0274	0.0281	0.0287
-1.8	0.0294	0.0301	0.0307	0.0314	0.0322	0.0329	0.0336	0.0344	0.0351	0.0359
-1.7	0.0367	0.0375	0.0384	0.0392	0.0401	0.0409	0.0418	0.0427	0.0436	0.0446
-1.6	0.0455	0.0465	0.0475	0.0485	0.0495	0.0505	0.0516	0.0526	0.0537	0.0548
-1.5	0.0559	0.0571	0.0582	0.0594	0.0606	0.0618	0.0630	0.0643	0.0655	0.0668
-1.4	0.0681	0.0694	0.0708	0.0721	0.0735	0.0749	0.0764	0.0778	0.0793	0.0808
-1.3	0.0823	0.0838	0.0853	0.0869	0.0885	0.0901	0.0918	0.0934	0.0951	0.0968
-1.2	0.0985	0.1003	0.1020	0.1038	0.1056	0.1075	0.1093	0.1112	0.1131	0.1151
-1.1	0.1170	0.1190	0.1210	0.1230	0.1251	0.1271	0.1292	0.1314	0.1335	0.1357
-1.0	0.1379	0.1401	0.1423	0.1446	0.1469	0.1492	0.1515	0.1539	0.1562	0.1587
-0.9	0.1611	0.1635	0.1660	0.1685	0.1711	0.1736	0.1762	0.1788	0.1814	0.1841
-0.8	0.1867	0.1894	0.1922	0.1949	0.1977	0.2005	0.2033	0.2061	0.2090	0.2119
-0.7	0.2148	0.2177	0.2206	0.2236	0.2266	0.2296	0.2327	0.2358	0.2389	0.2420
-0.6	0.2451	0.2483	0.2514	0.2546	0.2578	0.2611	0.2643	0.2676	0.2709	0.2743
-0.5	0.2776	0.2810	0.2843	0.2877	0.2912	0.2946	0.2981	0.3015	0.3050	0.3085
-0.4	0.3121	0.3156	0.3192	0.3228	0.3264	0.3300	0.3336	0.3372	0.3409	0.3446
-0.3	0.3483	0.3520	0.3557	0.3594	0.3632	0.3669	0.3707	0.3745	0.3783	0.3821
-0.2	0.3859	0.3897	0.3936	0.3974	0.4013	0.4052	0.4090	0.4129	0.4168	0.4207
-0.1	0.4247	0.4286	0.4325	0.4364	0.4404	0.4443	0.4483	0.4522	0.4562	0.4602
0.0	0.4641	0.4681	0.4721	0.4761	0.4801	0.4840	0.4880	0.4920	0.4960	0.5000

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986

Table 3: This table contains values of $\Pr(Z < z)$, where $Z \sim N(0, 1)$