# Chapter 8: Model solutions

1. Remember the three steps in formulating a linear programming problem:

   (a) Identify the **decision variables**

   (b) Identify the **constraints**

   (c) Form the **objective function**

   **Step 1: Decision variables**
   Going straight to the last sentence of the question, we can see that the quantities we need to know in order to solve the problem are 1) the number of type A machines to buy and 2) the number of type B machines to buy. Thus, our decision variables are

$$x = \text{number of type A machines bought} \qquad \text{and}$$
$$y = \text{number of type B machines bought.}$$

   **Step 2: Constraints**
   To help us identify the constraints, we could complete the table given in the question (also adding a column for profit if you like). Thus, we have

| | Number of operators | Floor space (m$^2$) | Profit (£) |
|---|---|---|---|
| Type A | 1 | 3 | 75 |
| Type B | 2 | 4 | 120 |
| Limits | 40 | 100 | |

   Considering the number of operators, since 1 type A machine requires 1 operator, then $x$ type A machines require $1 \times x = x$ operators. Similarly, since 1 type B machine requires 2 operators, $y$ type B machines require $2 \times y = 2y$ operators. Thus, the total number of operators is $x + 2y$. Since the maximum number of operators available is 40, the first constraint is

$$x + 2y \leq 40.$$

   Considering floor space, since 1 type A machine requires 3m$^2$ of floor space, $x$ type A machines require $3 \times x = 3x$ m$^2$ of floor space. Similarly, since 1 type B machine requires 4 m$^2$ of floor space, $y$ type B machines will require $4 \times y = 4y$ m$^2$ of floor space. Thus the total amount of floor space used is $(3x + 4y)$ m$^2$, and since the maximum amount of floor space available is 100 m$^2$, the second inequality is

$$3x + 4y \leq 100.$$

   Since we can't buy a negative number of each type of machine, we also have the following non–negativity constraints:

$$x \geq 0 \qquad \text{and}$$
$$y \geq 0.$$

**Step 3: Objective function**
Our objective is to maximise the profit, $P$. Using the last column in the table, we can see that from one type A machine we will get £75 profit, so from $x$ type A machines we will make $£(75 \times x) = £75x$ profit. Similarly, from one type B machine we will make £120 profit, so from $y$ type B machines we will make $£(120 \times y) = £120y$ profit. Thus, the total profit made will be $£(75x + 120y)$, and so our objective function is

$$P = 75x + 120y.$$

**So, to summarise, we have the following linear programming problem:**

Maximise $P = 75x + 120y$ subject to the following constraints:

$$\begin{aligned} x + 2y &\leq 40, \\ 3x + 4y &\leq 100, \\ x &\geq 0 \quad \text{and} \\ y &\geq 0. \end{aligned}$$

2. We can apply similar reasoning to this problem as used in question 1. Drawing up a table, we get:

| | Machine time (hours) | Craftsman's time (hours) | Profit (£) |
|---|---|---|---|
| Toy A | 3 | 2 | 10 |
| Toy B | 3 | 4 | 12 |
| Limits | 40 | 50 | |

**Step 1: Decision variables**
The decision variables are:

$$\begin{aligned} x &= \text{number of toy A's made} \quad \text{and} \\ y &= \text{number of toy B's made.} \end{aligned}$$

**Step 2: Constraints**
We have

$$\begin{aligned} 3x + 3y &\leq 40, \\ 2x + 4y &\leq 50, \\ x &\geq 0 \quad \text{and} \\ y &\geq 0. \end{aligned}$$

**Step 3: Objective function**
Our objective is to maximise the profit, $P$, where

$$P = 10x + 12y.$$

**Thus, to summarise, we have the following linear programming problem:**

Maximise $P = 10x + 12y$ subject to the following constraints:

$$\begin{aligned} 3x + 3y &\leq 40, \\ 2x + 4y &\leq 50, \\ x &\geq 0 \qquad \text{and} \\ y &\geq 0. \end{aligned}$$