Chapter 1 (*ctd*) exercises: Model solutions

Questions 1 and 2 were included in last week's exercises (page 12 of these notes). However, we have included them again here to provide a contrast against questions 3, 4 and 5 in which the population standard deviation is unknown.

1. We use the formulae

$$\begin{aligned} \bar{x} &\pm 1.96 \times \sqrt{\sigma^2/n} \qquad \text{and} \\ \bar{x} &\pm 2.58 \times \sqrt{\sigma^2/n} \end{aligned}$$

   to calculate the 95% and 99% confidence intervals (respectively). Now we know that

$$\begin{aligned} \bar{x} &= 750\text{g}, \\ \sigma^2 &= 100 \qquad \text{and} \\ n &= 50. \end{aligned}$$

   Thus, the 95% confidence interval for μ is found as

$$\begin{aligned} 750 &\pm 1.96 \times \sqrt{100/50} \qquad \text{i.e.} \\ 750 &\pm 1.96 \times \sqrt{2} \qquad \text{i.e.} \\ 750 &\pm 2.771859. \end{aligned}$$

   So, the 95% confidence interval for μ is (747.228g, 752.772g).

   Similarly, the 99% confidence interval for μ is found as

$$\begin{aligned} 750 &\pm 2.58 \times \sqrt{100/50} \qquad \text{i.e.} \\ 750 &\pm 2.58 \times \sqrt{2} \qquad \text{i.e.} \\ 750 &\pm 3.648671. \end{aligned}$$

   So the 99% confidence interval for μ is (746.351g, 753.649g).

2. The 95% confidence interval (as before) is found as

$$\bar{x} \pm 1.96 \times \sqrt{\sigma^2/n}.$$

   We have

$$\begin{aligned} \bar{x} &= 98\text{mm}, \\ \sigma^2 &= 50 \qquad \text{and} \\ n &= 100. \end{aligned}$$

   Thus, the 95% confidence interval is

$$\begin{aligned} 98 &\pm 1.96 \times \sqrt{50/100} \qquad \text{i.e.} \\ 98 &\pm 1.96 \times \sqrt{0.5} \qquad \text{i.e.} \\ 98 &\pm 1.385929. \end{aligned}$$

Thus, the 95% confidence interval for μ is (96.614mm, 99.386mm).

Since the confidence interval does not cover the target value of 100mm, we can say that the process is *not* satisfactory.

3. In this question, the population variance σ^2 is unknown, and so the 95% confidence interval is found as

$$\bar{x} \quad \pm \quad t \times \sqrt{s^2/n}.$$

We are given the sample mean and variance: $\bar{x} = 55$ and $s^2 = 100$, and the sample size, n is 40. Thus, to calculate the confidence interval, we only need to obtain t, which can be found from table 1.1. The degrees of freedom, $\nu = n - 1 = 40 - 1 = 39$, and so we use the ∞ row in table 1.1. For a 95% confidence interval, this gives a t value of 1.96, and so the interval is calculated as

$$\begin{aligned} 55 \quad &\pm \quad 1.96 \times \sqrt{100/40} \qquad \text{i.e.} \\ 55 \quad &\pm \quad 3.099. \end{aligned}$$

So the 95% confidence interval for μ is (51.9%, 58.1%).

4. In this question, σ^2 is again unknown. Thus, the 95% and 99% confidence intervals are given by

$$\bar{x} \quad \pm \quad t \times \sqrt{s^2/n}.$$

For the 95% confidence interval, and on $\nu = n - 1 = 12 - 1 = 11$ degrees of freedom, the t value can be found from table 1.1 as 2.201. So the 95% confidence interval is

$$\begin{aligned} 110 \quad &\pm \quad 2.201 \times \sqrt{220/12} \qquad \text{i.e.} \\ 110 \quad &\pm \quad 9.424. \end{aligned}$$

Thus, the 95% confidence interval for the population mean IQ is (100.58, 119.42).

For the 99% confidence interval, the corresponding t value is found to be 3.106. So the 99% confidence interval is

$$\begin{aligned} 110 \quad &\pm \quad 3.106 \times \sqrt{220/12} \qquad \text{i.e.} \\ 110 \quad &\pm \quad 13.299. \end{aligned}$$

Thus, the 99% confidence interval for the population mean IQ is (96.70, 123.30).

Only the 99% confidence interval captures the known population mean IQ of 100.

5. As with question 4, the population variance σ^2 is unknown. Thus, the 95% confidence interval is given by

$$\bar{x} \quad \pm \quad t \times \sqrt{s^2/n}.$$

From table 1.1, and on $\nu = n - 1 = 14 - 1 = 13$ degrees of freedom, the value of t for a 95% confidence interval is 2.160. However, unlike the previous questions, we don't know the values of $\bar{x}$ and s^2 in this question, and so we have to calculate these first! Now

$$\begin{aligned} \bar{x} &= \frac{10 + 12 + 15 + \ldots + 7}{14} \\ &= 10.928 \end{aligned}$$

and

$$\begin{aligned} s^2 &= \frac{(10 - 10.928)^2 + (12 - 10.928)^2 + \ldots + (7 - 10.928)^2}{13} \\ &= 10.071. \end{aligned}$$

Thus, the 95% confidence interval is found to be

$$\begin{aligned} 10.928 \quad &\pm \quad 2.160 \times \sqrt{10.071/14} \qquad \text{which is} \\ 10.928 \quad &\pm \quad 1.832. \end{aligned}$$

So, the 95% confidence interval is (9.096, 12.760). Since this interval does not cover the whole Northumbria police average of 8 cars per day, we can say that this speed camera is out of line with the rest of the area. In fact, there seems to be an above average number of cars caught speeding at this particular speed camera.

Note: A calculator in `Stats` mode can be used to compute the mean and variance.