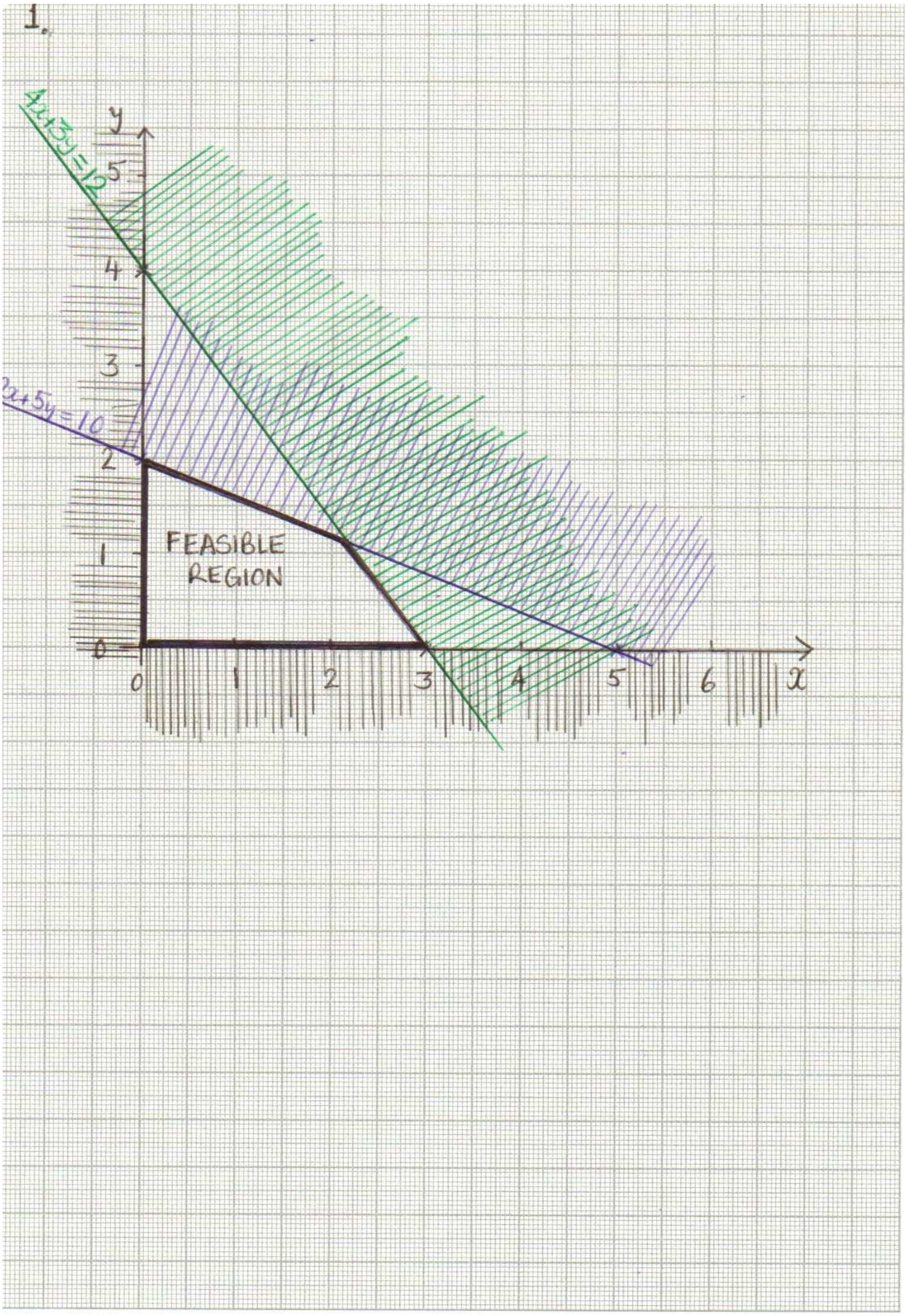
Chapter 10 – solutions to tutorial exercises



2. Question 1 in exercises 8 resulted in the following linear programming problem:

Maximise $P = 75x + 120y$ subject to

$$
\begin{aligned}
x + 2y &\leq 40, \\
3x + 4y &\leq 100, \\
x &\geq 0, \\
y &\geq 0,
\end{aligned}
$$

where

x = number of type A machines bought, and
y = number of type B machines bought;

P = Profit (£).

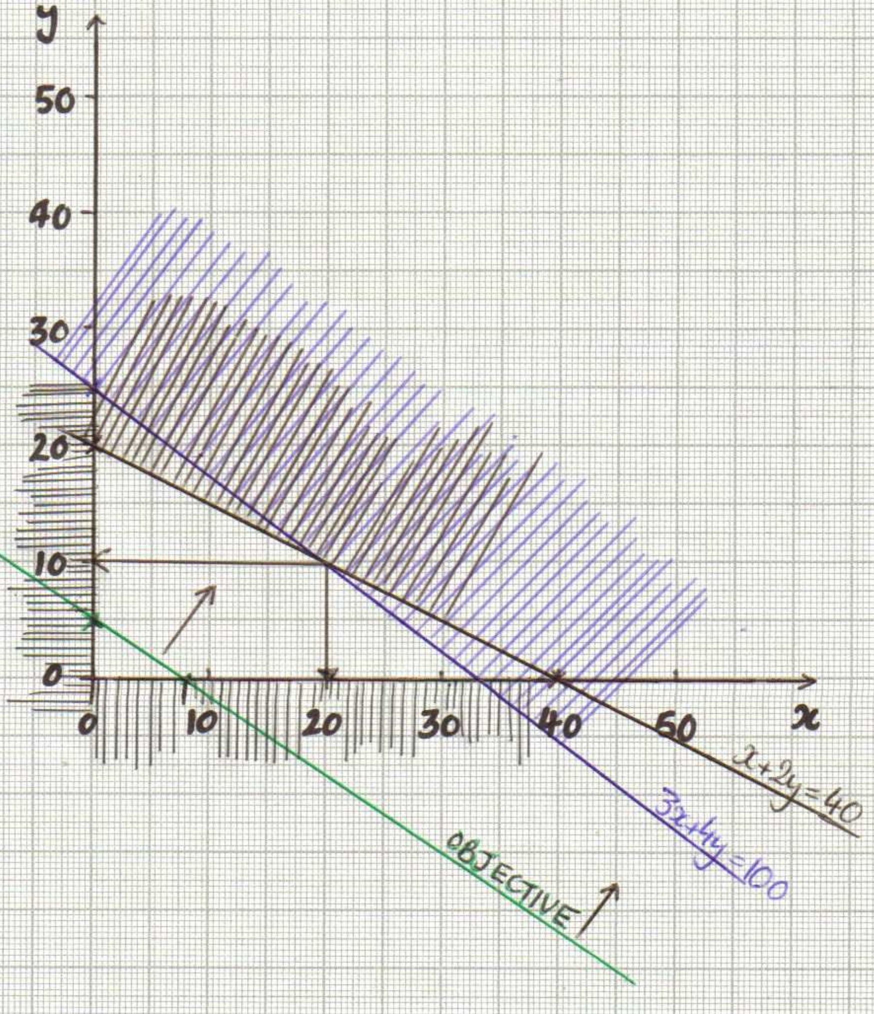
Consider $x + 2y \leq 40$.
For $x + 2y = 40$,
- when $x = 0$, $y = 20$
- when $y = 0$, $x = 40$.

Consider $3x + 4y \leq 100$.
For $3x + 4y = 100$,
- when $x = 0$, $y = 25$
- when $y = 0$, $x = 33.3$.

For the objective line, we have $75x + 120y = 600$.
- when $x = 0$, $y = 5$.
- when $y = 0$, $x = 8$.

From the graph, it is clear that we should buy 20 type A machines and 10 type B machines, giving a profit of

$$
\begin{aligned}
P &= 75 \times 20 + 120 \times 10 \\
&= £2700.
\end{aligned}
$$



y
50
40
30
20
10
0
0
10
20
30
40
50
x
x+2y=40
3x+4y=100
OBJECTIVE

3.

Toy	Material (m^2)	Time (mins)	Profit (£)
Bear	5	12	1·50
Cat	8	8	1·75
Limits	2000	2880 (mins!)	

(i) STEP 1: DECISION VARIABLES

Let x : number of bears made, and
y : number of cats made.

STEP 2: CONSTRAINTS

$$5x + 8y \leq 2000 \quad \text{(material)}$$
$$12x + 8y \leq 2880 \quad \text{(time)}$$

$$\left.\begin{array}{l} x \geqslant 0 \\ y \geqslant 0 \end{array}\right\} \text{non-negativity constraints.}$$

STEP 3: OBJECTIVE FUNCTION

Let P = profit. Then

$$P = 150x + 175y \quad \text{(in pence).}$$

(ii) For the diagram,

$5x+8y \leq 2000$
- when $x=0$, $y=250$
- when $y=0$, $x=400$

$12x+8y \leq 2880$
- when $x=0$, $y=360$
- when $y=0$, $x=240$.

OBJECTIVE LINE
$P = 150x + 175y$

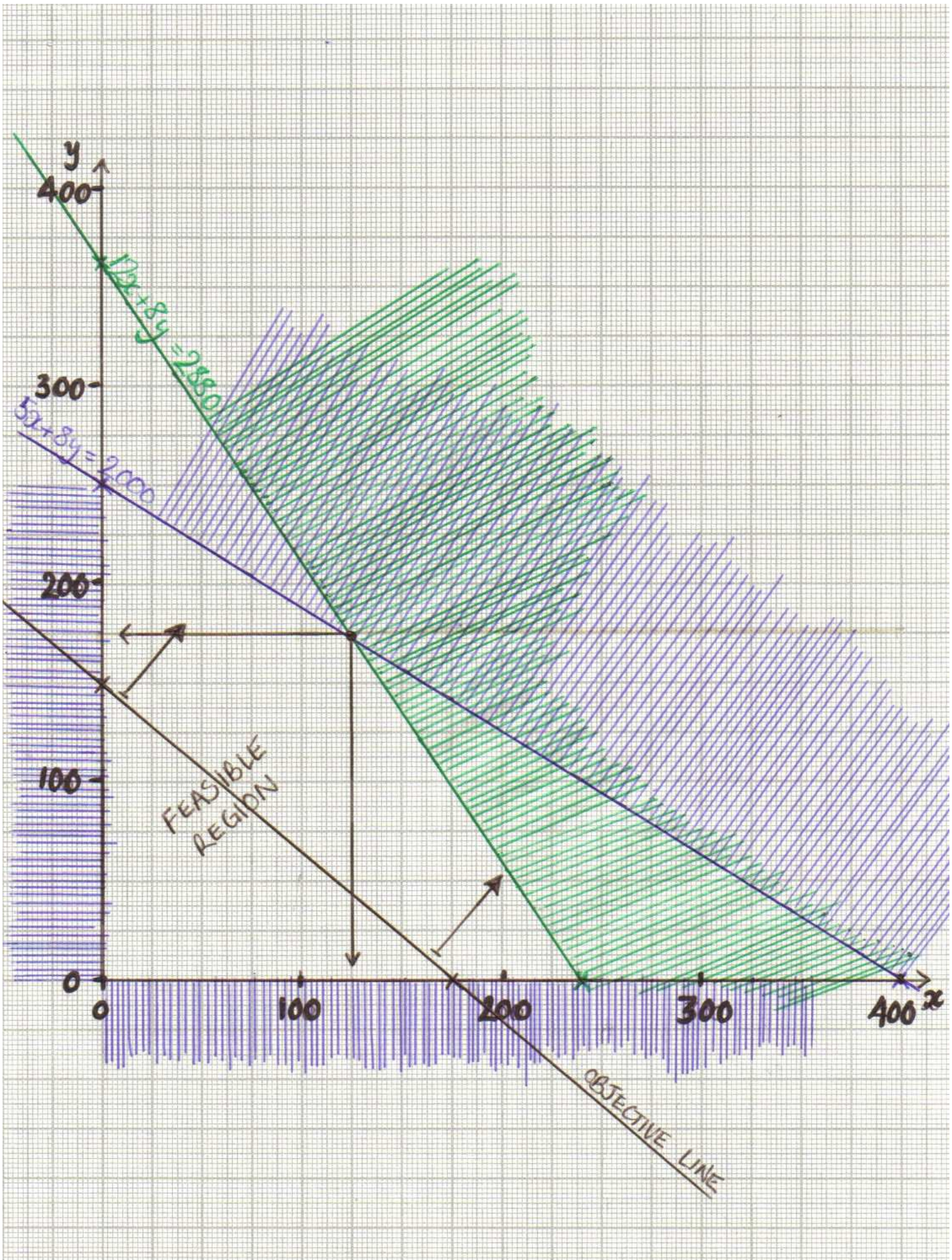
Try $P = 150 \times 175 = 26250$.
- when $x=0$, $y=150$
- when $y=0$, $x=175$.

Looking at the graph, we see that the maximum profit is achieved at the intersection between the lines $12x+8y=2880$ and $5x+8y=2000$.

An accurate graph will give $x = 125.7$ and $y = 171.4$. So we should make 125 bears and 171 cats to maximise our profit.

(iii) The maximum profit is thus

$$\begin{aligned} P &= 150 \times 125 + 175 \times 171 \\ &= 48675p \\ &= £486.75. \end{aligned}$$



y
400
300
200
100
0
0
100
200
300
400
x
12x + 8y = 2880
5x + 8y = 2000
FEASIBLE REGION
OBJECTIVE LINE