# Chapter 1 exercises: Model solutions

1. We use the formulae

$$\begin{aligned} \bar{x} &\pm 1.96 \times \sqrt{\sigma^2/n} \qquad \text{and} \\ \bar{x} &\pm 2.58 \times \sqrt{\sigma^2/n} \end{aligned}$$

to calculate the 95% and 99% confidence intervals (respectively). Now we know that

$$\begin{aligned} \bar{x} &= 750\text{g}, \\ \sigma^2 &= 100 \qquad \text{and} \\ n &= 50. \end{aligned}$$

Thus, the 95% confidence interval for $\mu$ is found as

$$\begin{aligned} 750 &\pm 1.96 \times \sqrt{100/50} \qquad \text{i.e.} \\ 750 &\pm 1.96 \times \sqrt{2} \qquad \text{i.e.} \\ 750 &\pm 2.771859. \end{aligned}$$

So, the 95% confidence interval for $\mu$ is (747.228g, 752.772g).

Similarly, the 99% confidence interval for $\mu$ is found as

$$\begin{aligned} 750 &\pm 2.58 \times \sqrt{100/50} \qquad \text{i.e.} \\ 750 &\pm 2.58 \times \sqrt{2} \qquad \text{i.e.} \\ 750 &\pm 3.648671. \end{aligned}$$

So the 99% confidence interval for $\mu$ is (746.351g, 753.649g).

2. The 95% confidence interval (as before) is found as

$$\bar{x} \pm 1.96 \times \sqrt{\sigma^2/n}.$$

We have

$$\begin{aligned} \bar{x} &= 98\text{mm}, \\ \sigma^2 &= 50 \qquad \text{and} \\ n &= 100. \end{aligned}$$

Thus, the 95% confidence interval is

$$\begin{aligned} 98 &\pm 1.96 \times \sqrt{50/100} \qquad \text{i.e.} \\ 98 &\pm 1.96 \times \sqrt{0.5} \qquad \text{i.e.} \\ 98 &\pm 1.385929. \end{aligned}$$

Thus, the 95% confidence interval for $\mu$ is (96.614mm, 99.386mm).

Since the confidence interval does not cover the target value of 100mm, we can say that the process is *not* satisfactory.

3. This question is based on your own random sample and so the answers will vary – please consult your notes from the tutorial.