

**Durham/Newcastle Statistics Graduate Course**  
**Design of Experiments**  
**March 2012**

Exercise Sheet 2

| Unit | Outcome | Rx | Other |
|------|---------|----|-------|
| 1    | 7.8     | 0  | 1     |
| 2    | 15.3    | 1  | 1     |
| 3    | 18.5    | 2  | 1     |
| 4    | 6.2     | 0  | 2     |
| 5    | 11.3    | 1  | 2     |
| 6    | 24.2    | 2  | 2     |
| 7    | 11.2    | 0  | 3     |
| 8    | 17.4    | 1  | 3     |
| 9    | 29.8    | 2  | 3     |
| 10   | 1.8     | 0  | 4     |
| 11   | 3.3     | 1  | 4     |
| 12   | 7.1     | 2  | 4     |

The above table gives the melanin concentration (outcome) measured in twelve skin samples following treatment with different doses of a chemical (OAG) labelled 0,1 and 2. For the moment ignore the column headed 'Other'.

Once you have entered the data into R, however you wish, please save them as they are needed in Sheet 3. You should also save quantities such as projection matrices, as some of these will also be needed in Sheet 3.

- Q1** Analyse these data using the matrix handling facilities in R to obtain the matrices  $P_0$ ,  $P_T$ ,  $P_W$ ,  $P_E$  and the vectors  $P_0y$ ,  $P_Wy$  and  $P_Ey$ . Show explicitly that the Sums of Squares add up.
- Q2** Take the opportunity to check the eigenvalues and traces of these matrices and their ranks ('`help(qr)`' may help). Also check formulae such as  $y^T P_W y = \|P_W y\|^2$
- Q3** Analyse these data using the usual '`lm()`' command and compare.
- Q4** Compute some of the products of pairs of the projection matrices and comment.