Durham/Newcastle Statistics Graduate Course
Design of Experiments
March 2012

Exercise Sheet 1

Q1 Prove the results concerning projections quoted in the lecture, namely

- **(i)** $P_T = P_T^T$
- **(ii)** $P_T^2 = P_T$
- **(iii)** The eigenvalues of P_T are all 0 or 1.
- **(iv)** $\mathsf{tr}(P_T) = \dim V_T$

Note: try to do these without assuming an explicit matrix form for P_T

Q2 Using the result for $\mathsf{var}(\hat{\tau}_E)$ in lectures, namely $\sigma^2 R^{-1}$, find the variance of $c^T \hat{\tau}_E$ for any given T-dimensional vector c.

Q3 An experiment to compare T treatments is designed in one of two ways.

- **(i)** All comparisons between the treatments are of interest;
- **(ii)** One treatment, say treatment 1, is a control treatment and interest focusses on comparisons with this treatment.

The replication of treatment j is r_j and the number of treatment applications, $N = \sum r_j$ is fixed. Using the result in Q2, how should you choose the r_j in the two cases?

Q4 If the vector subspace W is the range of a matrix X the orthogonal projection onto the range of X is well know to be $X(X^T X)^{-1} X^T$. Now assuming this formula, show that the $\hat{\tau}_j$ derived in lectures are of a familiar and sensible form.