Gradient, Divergence and Curl

Please hand in solutions on Monday 5 December, 2011

To earn full marks, your solutions must be correct, complete, and presented in a logical form using clear, unambiguous and consistent mathematical notation. In particular, marks will be deducted for not using correct notation for vectors in handwriting (e.g. $\underline{r}$, or $\utilde{r}$, or $\vec{r}$). A collection of isolated formulae, without their logical sequence being indicated with equality signs (or any other appropriate symbol), and without brief reasoning being supplied where necessary, will not be considered as a complete solution.

Note that no help will be given (in the Drop-in, or elsewhere) for the *starred* questions, Questions 2(b) and 5(c).

Question 1

Find the directional derivative of the scalar field $f(x, y, z) = xyz$ in the direction of $\mathbf{b} = (0, 4, 3)$, at the point $\mathbf{r} = (-2, 3, 4)$.

Question 2

(a) Expand and simplify $\nabla \cdot \big(f(r)\mathbf{r}\big)$, where $r = |\mathbf{r}|$, $\mathbf{r} = (x, y, z)$ and f is a differentiable scalar field.

(b) $(*)$ Find $f(r)$ such that $\nabla \cdot \big(f(r)\mathbf{r}\big) = 0$.

Question 3

Expand and simplify $\nabla \times (r\mathbf{r})$, where $r = |\mathbf{r}|$ and $\mathbf{r} = (x, y, z)$.

Question 4

Show that $\nabla \times (f\mathbf{v}) = \nabla f \times \mathbf{v} + f\nabla \times \mathbf{v}$, where $f(x, y, z)$ is a differentiable scalar field and $\mathbf{v}(x, y, z)$ is a differentiable vector field.

Question 5

Given that $\mathbf{c} = (c_x, c_y, c_z)$ is a constant vector, and $\mathbf{r} = (x, y, z)$ is the position vector, simplify the following expressions, expressing your answer in terms of $\mathbf{c}$ wherever possible:

(a) $\nabla(\mathbf{c} \cdot \mathbf{r})$; **(b)** $\mathbf{c}(\nabla \cdot \mathbf{r})$; **(c)** $(*)$ $\nabla \times (\mathbf{c} \times \mathbf{r})$.

GRS **21/11/2010**