MAS2104/MAS3104 **Exercises, Part 3**

Surfaces and Normal Vectors. The Gradient

Please hand in solutions on Monday 21 November, 2011

To earn full marks, your solutions must be correct, complete, and presented in a logical form using clear, unambiguous and consistent mathematical notation. In particular, marks will be deducted for not using correct notation for vectors in handwriting (e.g. $\underline{r}$, or $\underset{\sim}{r}$, or $\vec{r}$). A collection of isolated formulae, without their logical sequence being indicated with equality signs (or any other appropriate symbol), and without brief reasoning being supplied where necessary, will not be considered as a complete solution.

Note that no help will be given (in the Drop-in, or elsewhere) for the *starred* questions, Questions 4(b) and 5(b).

Question 1

Consider the surfaces

(a) $(u, v) \mapsto (v\cos u, v\sin u, v^2),\ 0 \le u \le 2\pi,\ 0 \le v \le 1$,

(b) $(u, v) \mapsto (2v\cos u, v\sin u, v),\ 0 \le u \le 2\pi,\ 0 \le v \le 1$.

For each surface, give an equation for the surface in each of the two forms **(i)** $f(x, y, z) = 0$ and **(ii)** $z = g(x, y)$. From *each* of these forms, find a normal vector and the corresponding unit normal vector.

For each of the surfaces (a) and (b), sketch the surface.

Question 2

The surfaces $z = \sqrt{x^2 + y^2}$ and $x^2 + y^2 + z^2 = 1\,,\ z \ge 0$ intersect along a horizontal circle. Find an equation describing this circle. The angle between their normals at the intersection points never changes. What is this angle?

Hint: An intersection line of two surfaces, $f(x, y, z) = 0$ and $g(x, y, z) = 0$, is obtained as a solution of two simultaneous equations, $f(x, y, z) = 0\,,\ g(x, y, z) = 0$.

Question 3

If f and g are two differentiable functions of x, y and z, show that $\nabla(fg) = f\nabla g + g\nabla f$.

Question 4

(a) Find ∇f for the cases **(i)** $f(x, y, z) = x + z$, **(ii)** $f(x, y, z) = x^2 + z^2$.

(b) $(*)$ For the two cases above, comment on how the direction of the vector ∇f varies in space.

Hint: Consider ∇f on the surfaces given by $f(x, y, z) =$ constant.

Question 5

Find **(a)** ∇r^2 , **(b)** $(*)$ $\nabla f(r)$, where $r = |\mathbf{r}|$, $\mathbf{r} = (x, y, z)$ and $f(r)$ is a differentiable function of r. Express your answers in terms of $\mathbf{r}$.

GRS **7/11/2011**