

## Curves in Two Dimensions

Please hand in solutions on Monday 24 October, 2011

To earn full marks, your solutions must be correct, complete, and presented in a logical form using clear, unambiguous and consistent mathematical notation. In particular, marks will be deducted for not using correct notation for vectors in handwriting (e.g.  $\underline{r}$ , or  $\underline{r}$ , or  $\vec{r}$ ). A collection of isolated formulae, without their logical sequence being indicated with equality signs (or any other appropriate symbol), and without brief reasoning being supplied where necessary, will not be considered as a complete solution.

Note that no help will be given (in the Drop-in, or elsewhere) for the *starred* question, Question 1(c).

### Question 1

For each of the following curves, given in parametric form, derive an expression for the curve in (explicit or implicit) geometric form. Sketch the curves and indicate with an arrow the direction in which the curves are traversed as  $t$  increases [ $t$  is in radians in (c) and (d)]:

- (a)  $t \mapsto (t^3, t)$ ,  $-1 \leq t \leq 1$ ;      (b)  $t \mapsto (t, t^{3/2})$ ,  $1 \leq t \leq 2$ ;  
(c) (\*)  $t \mapsto (t^2, \sin t)$ ,  $0 \leq t \leq 3$ ;      (d)  $t \mapsto (3 \cos t, -3 \sin t)$ ,  $-\frac{1}{2}\pi \leq t \leq \frac{1}{2}\pi$ ;  
(e)  $t \mapsto (1, 2t)$ ,  $0 \leq t \leq 1$ .

### Question 2

For each of the curves (a)–(e) in question 1, calculate the tangent vector  $\mathbf{v}$ , and identify all points (if any) where the tangent is vertical (i.e. purely in the  $y$ -direction).